Supercharge cohomology in holographic theories

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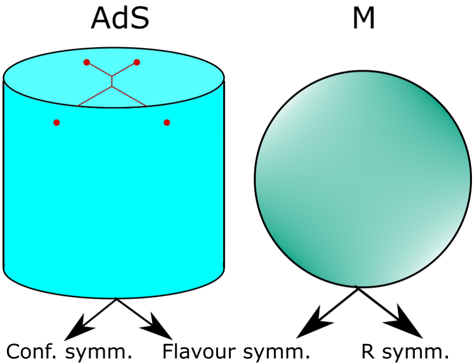
See [2512.23603] with L. Pipolo de Gioia

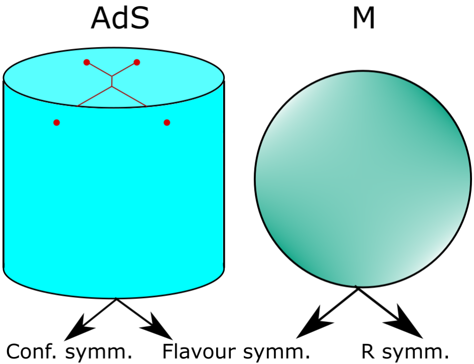
Benefits of the AdS/CFT correspondence



$\mathcal{N} = 4$ Super Yang-Mills dual to Type IIB on $AdS_5 \times S^5$ [Maldacena; 1997] .

ABJM theory dual to Type IIA on $AdS_4 \times \mathbb{CP}^3$ [Aharony, Bergman, Maldacena, Jafferis; 2008] .

Weak-strong duality: $G_N L^{1-d} \sim \frac{1}{N}$ and $\left(\frac{L}{\ell_s}\right)^{\#} \sim \lambda$.



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Black holes radiate like an object with $T = \frac{1}{8\pi GM}$ [Hawking; 1975] .

Is this process unitary? How do we interpret the associated microstates?

Microstates in an SCFT

Encode the spectrum of a superconformal theory with

$$Z(w, x, y) = \mathrm{Tr}\left[w^E \prod_m x_m^{J_m} \prod_n y_n^{R_n} \right].$$

E	Scaling dimension
J_m	Lorentz spins
R_n	R-symmetry charges

Suppose Q, raises E by $\frac{1}{2}$, lowers J_0 by $\frac{1}{2}$ and is only charged under R_0.

Then $Q^\dagger$, lowers E by $\frac{1}{2}$, raises J_0 by $\frac{1}{2}$ and is only charged under R_0.

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$$I(w,x,y) = \mathrm{Tr}\left[(-1)^F w^{E+J_0} \prod_{m\neq 0} x_m^{J_m} \prod_{n\neq 0} y_n^{R_n} \right]$$

- $|\psi\rangle$ only killed by Q? $\Rightarrow |\psi\rangle, Q^\dagger |\psi\rangle$ cancel
- $|\psi\rangle$ only killed by $Q^\dagger$? $\Rightarrow |\psi\rangle, Q |\psi\rangle$ cancel
- $|\psi\rangle$ killed by neither? $\Rightarrow |\psi\rangle, Q|\psi\rangle, Q^\dagger|\psi\rangle, QQ^\dagger|\psi\rangle$ all cancel

Insights from the index

The superconformal index is a signed counting of BPS states (elements of $\ker Q \cap \ker Q^\dagger$) and it is independent of the coupling [Kinney, Maldacena, Minwalla, Raju; 2005] .

$$I_N(w,1,1) = \sum_n d_N(n) w^n$$

$$\log|d_\infty(n)| \sim \frac{\pi}{3}\sqrt{5n}, \quad \log|d_N(cN^2)| \sim \frac{\pi N^2 c^{2/3}}{2\cdot 3^{1/6}}$$

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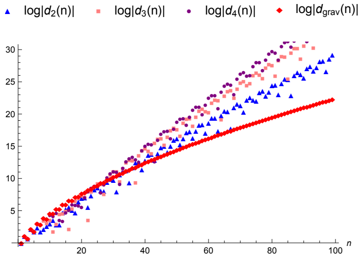

In $\mathcal{N}=4$ SYM, double scaled limit counts enough states to match BPS black holes with $S = \frac{A}{4G_N} \sim N^2$ [Cabo-Bizet, Cassani, Martelli, Murthy; 2018] [Murthy; 2020] .

Many states are left over after recombination

$$L\bar{L}[j;\bar{\jmath}]^{(R_1,R_2,R_3)}_\Delta \to A_1\bar{L}[j;\bar{\jmath}]^{(R_1,R_2,R_3)}_\Delta \oplus A_1\bar{L}[j-1;\bar{\jmath}]^{(R_1+1,R_2,R_3)}_{\Delta+1/2}.$$

Cohomological viewpoint

Theorem: When Q is a nilpotent supercharge, each Q-cohomology class contains exactly one BPS state.

$$\left[Q_0 + \lambda Q_1 + \dots \right] \left[|\psi_0\rangle + \lambda |\psi_1\rangle + \dots \right] = 0$$
$$(*) Q_1 |\psi_0\rangle = -Q_0 |\psi_1\rangle$$

Failing to be closed and failing to not be exact happen together.
- $|\psi_0\rangle$ is manifestly a cohomology class at $O(\lambda^0)$ and (*) obstruction says it is not closed at $O(\lambda^1)$.
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Most cohomology classes in a holographic theory are not stable with N. When building gauge invariant candidates for $|\psi_0\rangle, |\psi_1\rangle, \dots$ out of fields in a Lagrangian, they could be either monotone or fortuitous [Chang, Lin; 2024] .

Holographic coverings

Definition: A holographic covering consists of a vector space $\widetilde{\mathcal{H}}$ and a sequence of ideals $I_{N+1} \subset I_N$ with a short exact sequence

$$0 \to I_N \xrightarrow{\imath_N} \widetilde{\mathcal{H}} \xrightarrow{\pi_N} \mathcal{H}_N \to 0.$$

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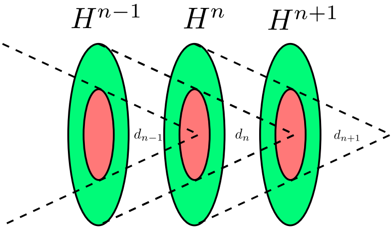
This implies $\mathcal{H}_N = \widetilde{\mathcal{H}}/I_N$. It also induces the long exact sequence

$$\dots \to H^n(I_N) \xrightarrow{\imath^*} H^n(\widetilde{\mathcal{H}}) \xrightarrow{\pi^*} H^n(\mathcal{H}_N) \xrightarrow{f} H^{n+1}(I_N) \to \dots$$

where monotone states are in $\operatorname{im}\pi^* = H^*(\widetilde{\mathcal{H}})/\operatorname{im}\imath^*$ and fortuitous states are in $\operatorname{im} f = H^*(\mathcal{H}_N)/\operatorname{im}\pi^*$.



Algebraic structures

A classically BPS operator hit by Q should be a cochain on some algebra — $\mathfrak{g}_N$ valued polynomials in some generators.

Associative algebra like $\mathbb{C}[z_\pm, \theta_{1,2,3}]$

$$H^*(\mathfrak{g}_N[A], \mathfrak{g}_N)$$

Ensures gauge invariance

Relative Lie-algebra cohomology.

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Relative dg-L_∞-algebra cohomology.

The connecting homomorphism f defining fortuity also appears in the distinguished triangle of a triangulated category.

$$I_N \xrightarrow{\imath^*} \widetilde{\mathcal{H}} \xrightarrow{\pi^*} \mathcal{H}_N \xrightarrow{f} I_N[1]$$

The twisted theory

The cohomology comes from a sector of the theory which is holomorphic (or holomorphic-topological).

$$\sigma^{\mu}_{-\dot{\alpha}} P_{\mu} = \{Q, \bar{Q}_{\dot{\alpha}}\}$$

Since forms are alternating under $\wedge$ just like Grassmann numbers, we can use BV superfields to write a nice action.

$$C(z, d\bar{z}) = c(z) + A_{\dot{\alpha}}(z) d\bar{z}^{\dot{\alpha}} + b(z) d\bar{z}^2, \quad \bar{\partial} = d\bar{z}^{\dot{\alpha}} \frac{\partial}{\partial \bar{z}^{\dot{\alpha}}}$$

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Singular OPEs in this theory are Q-exact so multiplication is well defined. Multiplying cohomology classes at $N \to \infty$ to study the BPS Hilbert space is valid even along an RG flow

[Budzik, Gaiotto, Kulp, Williams, Wu, Yu; 2023] .

Applications

First fortuitous operator in 4d $\mathcal{N} = 4$ SYM with $G = SU(2)$

[Chang, Lin; 2022] [Choi, Kim, Lee, Park; 2022]

$$\mathcal{O} = \epsilon^{m_1 m_2 m_3} Tr(\phi^{m_4}\psi_{m_1}) Tr(\phi^{m_5}\psi_{m_2}) Tr(\psi_{m_3}[\psi_{m_4}, \psi_{m_5}]) + Q(\dots).$$

Infinitely many more were built from BMN fields (ϕ^m, ψ_m, f) in [Choi, Kim, Lee, Lee, Park; 2023] and they satisfy

$$\mathrm{Tr}(X_1)\dots\mathrm{Tr}(X_L)\mathcal{O}' = Q(\dots) \quad \textit{unless} \quad X_* = \phi^m f - \frac{1}{4}\epsilon^{mnp}\psi_n\psi_p.$$

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First fortuitous operator in $U(2)_k \times U(2)_{-k}$ ABJM

[Belin, Singh, Vadala, Zaffaroni; 2025] [CB, de Gioia; 2025]

$$\mathcal{O}^a_b = \epsilon^{cd}\left[4\,Tr(\bar{\psi}_c\phi_d\bar{\phi}^a\phi_b) - 3\,Tr(\bar{\psi}_c\phi_d)Tr(\bar{\phi}^a\phi_b)\right] - c.c. + Q(\dots).$$

Bilinears mimic the BMN fields and no-hair relations appear here too.

$$\text{Tr}(\bar{\phi}^{\{a}\phi_b)\mathcal{O}^{c\}}_d\epsilon^{bd} = 0$$

Future directions

- The monotone/fortuitous dichotomy motivates a renewed look at index technology.
- Cohomology classes in one theory form a good starting point for understanding another — deformations of the differential are highly constrained.
- There is also a notion of deforming the algebra — one can obtain A_∞ algebras from a quasi-isomorphism.
- An interesting theory to look at would be $\mathcal{N} = 2$ superconformal QCD with $G = USp(2N)$. This has four fundamental hypers and therefore an $SO(8)$ flavour symmetry [Seiberg, Witten; 1994] .
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Thanks for your attention!