New tools for conformal manifolds in one and two dimensions

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Conformal field theory

More symmetric than a “typical” QFT:

Translations	$x'_\mu = x_\mu + a_\mu$
Rotations	$x'_\mu = \Lambda^\nu_\mu x_\nu$
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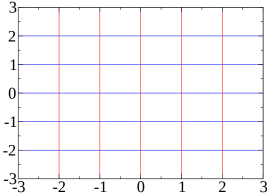
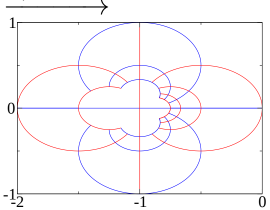


$b_\mu = (1,0)$



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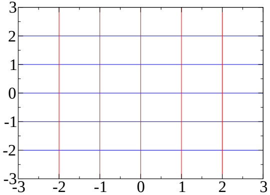

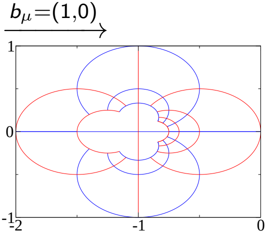

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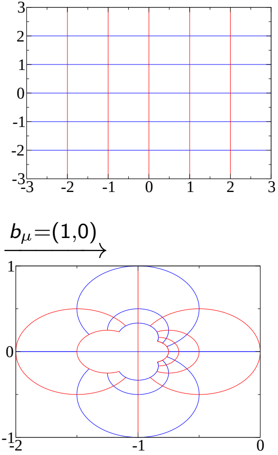

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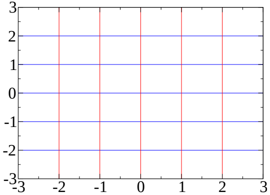

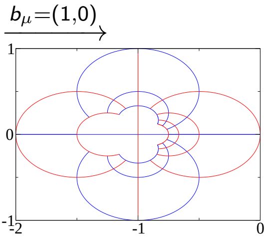

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- $\langle \phi(x_1)\phi(x_2) \rangle = \frac{1}{|x_{12}|^{2\Delta}}$
- $\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3) \rangle = \frac{\lambda_{123}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}|x_{13}|^{\Delta_1+\Delta_3-\Delta_2}}$
- $\phi_1(x_1)\phi_2(x_2) = \sum_\mathcal{O} \frac{\lambda_{12\mathcal{O}}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta}} C_{(\mu)}(x_{12}, \partial_2)\, \mathcal{O}^{(\mu)}(x_2)$

Conformal bootstrap

$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4) \rangle$ includes unknown function of $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$ and $v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$ but there are two ways to compute it!

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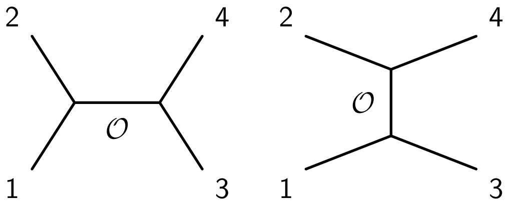

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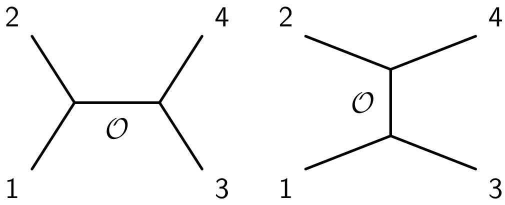

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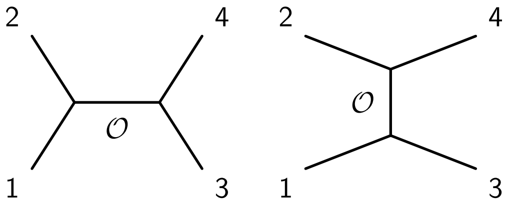

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Sum over $\mathcal{O}$ must include operatos in certain ranges for crossing symmetry to have a solution. [Rattazzi, Rychkov, Tonni, Vichi; 0807.0004]

$$\sum_{\mathcal{O}} \lambda_{\phi\phi\mathcal{O}}^2 \left[v^{\Delta_\phi} G_{\mathcal{O}}(u,v) - u^{\Delta_\phi} G_{\mathcal{O}}(v,u) \right] = 0$$

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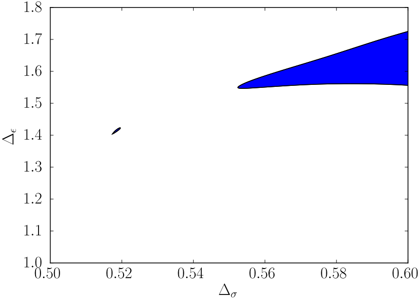

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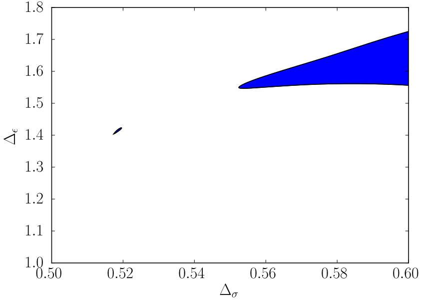

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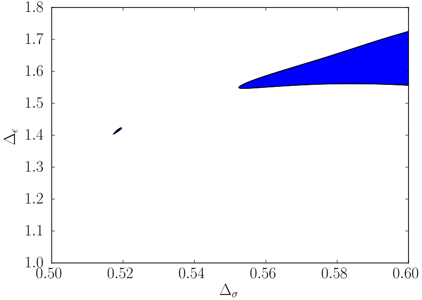

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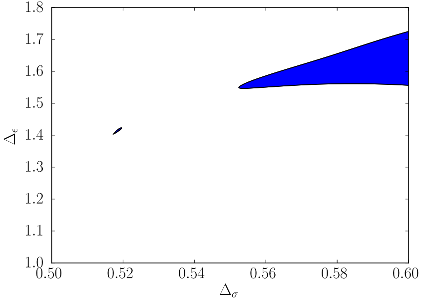

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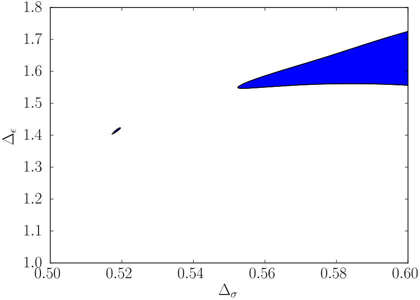

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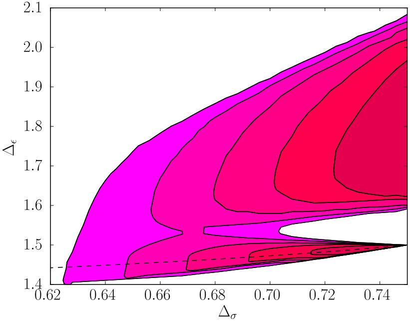

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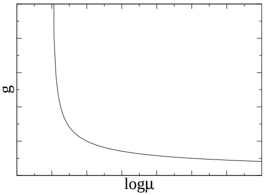

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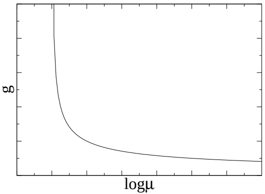

Coupling runs according to $\beta(g)$,

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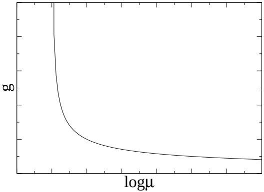

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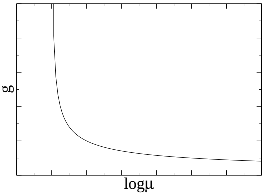

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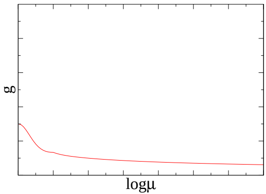

Make β_1 cancel β_0 by picking g_*.

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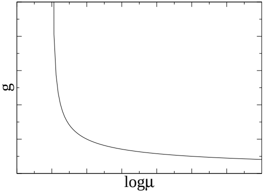

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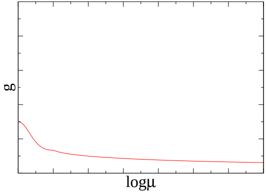

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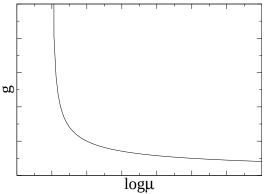

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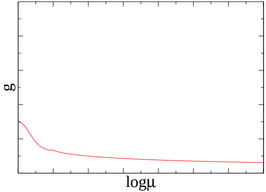

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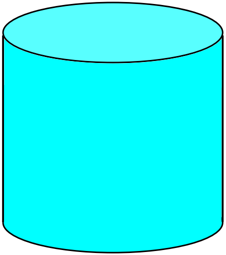

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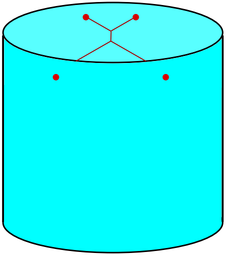

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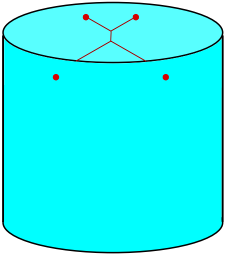

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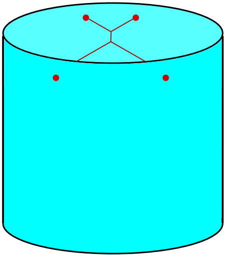
$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right)$$



$$(mL)^2 = \Delta(\Delta - d)$$

$\Delta = d$ from symmetry breaking.

[Carmi, Di Pietro, Komatsu; 1810.04185]

Conformal manifolds

Special theories have $\beta(g) = 0$ for any g.

$$S = \frac{1}{4\pi\alpha'} \int_{\mathbb{R}^2} G^{\mu\nu}(X) \partial X_\mu \cdot \partial X_\nu d\sigma d\tau$$

Conformality requires $R_{\mu\nu} = 0$ but there could be moduli.

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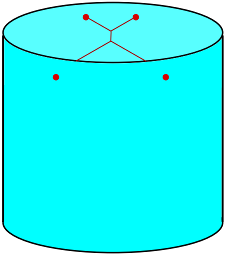

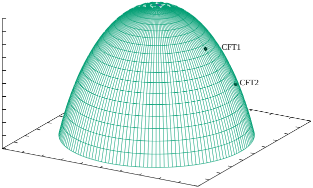

Flow by adding $\int \delta g \hat{O} dx$

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SYK-like models

$$S = \int \frac{1}{2}\chi_i \frac{d}{d\tau}\chi^i + \frac{1}{4!} J_{ijkl}\chi^i\chi^j\chi^k\chi^l d\tau \; , \; \left\langle J_{ijkl}J^{ijkl} \right\rangle = 3! J^2 / N^3$$

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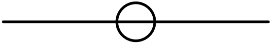

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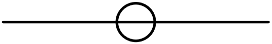

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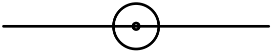

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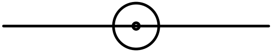

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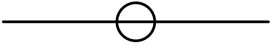

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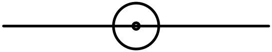
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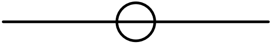
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Kinetic term $\mapsto \int \mathrm{sgn}(\tau_{12})\chi_i(\tau_1)\chi^i(\tau_2)\frac{d\tau_1 d\tau_2}{|\tau_{12}|^{\frac{3}{2}}}$. [Gross, Rosenhaus; 1706.07015]

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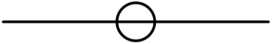
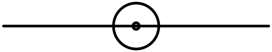
 $= O(1)$  $= O(1)$

 $= O(1)$  $= O\left(\frac{1}{N}\right)$

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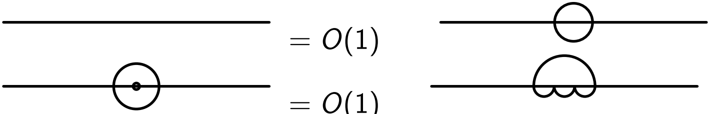

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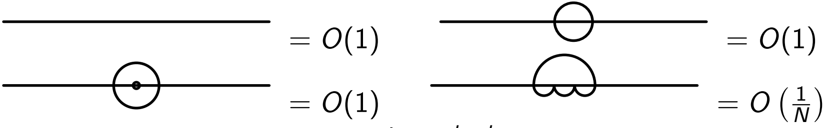

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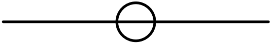
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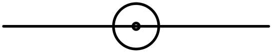
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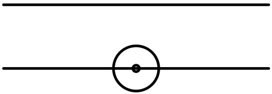

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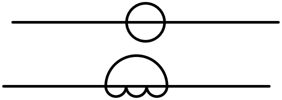

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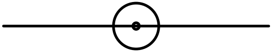

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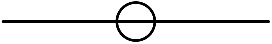

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Free boson with a boundary deformation

Pieces of $\Phi(z,\bar{z}) = X(z) + \bar{X}(\bar{z})$ agree on Neumann boundary.

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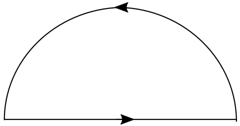

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If $\Phi \sim \Phi + 2\pi R = \Phi + 2\sqrt{2}\pi N$, the spectrum contains:

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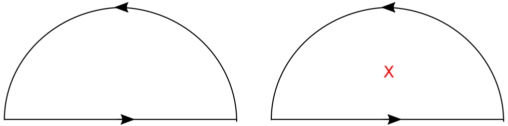

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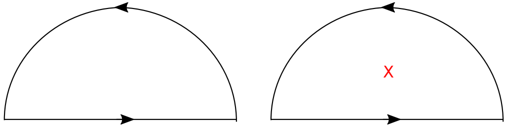

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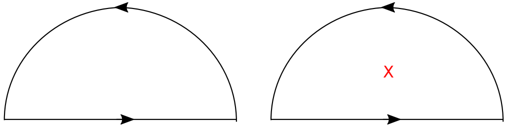

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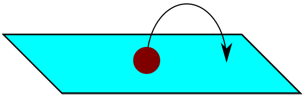
$$Z_{[0,L]\times[0,T]} = \left\langle B \right| e^{-\frac{2\pi T}{L} H} \left| B \right\rangle$$

[Callan, Klebanov, Ludwig, Maldacena; hep-th/9402113]

Instantons (with Rastelli, Roumpedakis)



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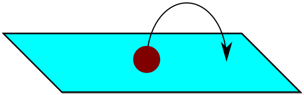

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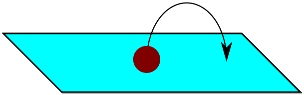

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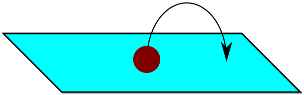

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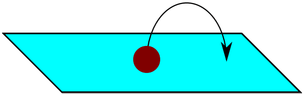

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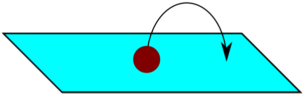
[Billo, Frau, Fucito, Lerda, Liccardo, Pesando; hep-th/0211250]



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Root vectors of $SO(8)$ appear at $R = \sqrt{2}$!

$$V_w + V_{\bar{w}} = \exp\left[i\alpha \cdot (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6)\right]$$

Conclusions

- Several theories can benefit from more bootstrap-like approaches.
- A system of differential equations controls the data along a conformal manifold.
- Promising targets include SYK-like models, BCFTs in $d = 1 + 1$ and the D-instanton.
- Non-linear sigma models, $\mathcal{N} = 4$ SYM and AdS Goldstones could be next.