Simplifying Plasma Balls and Black Holes with Nonlinear Diffusion

Connor Behan

July 14, 2014

What is universal in AdS / CFT?

String theories (with CFT duals) form an infinite family:

- $AdS_5 \times \mathbb{S}^5$
- $AdS_4 \times \mathbb{CP}^3$
- $AdS_3 \times \mathbb{S}^3 \times \mathbb{T}^4$
- ...

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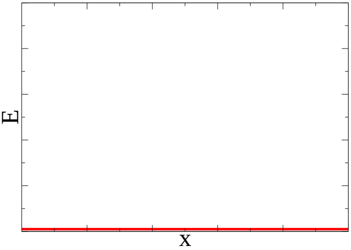
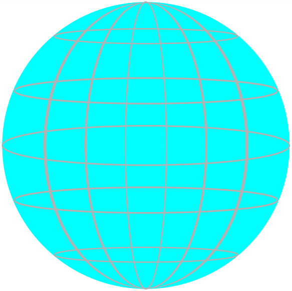
First look at $AdS_5 \times \mathbb{S}^5 \Leftrightarrow \mathcal{N} = 4$ Super Yang-Mills.

Gauge theory with one scale





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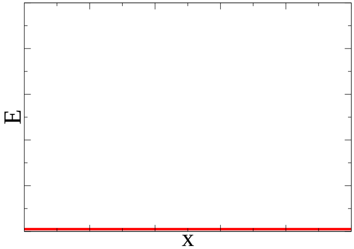

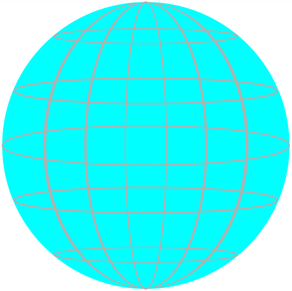

$$ds^2 = -\left(1+\frac{r^2}{L^2}\right)dt^2 + \left(1+\frac{r^2}{L^2}\right)^{-1}dr^2 + r^2 d\Omega_3^2$$

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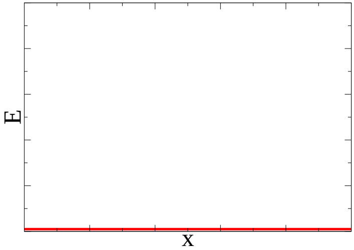

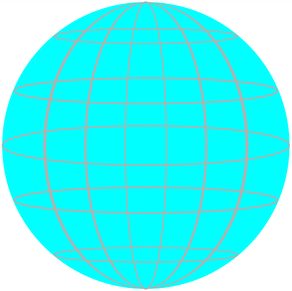

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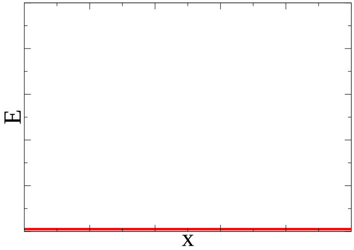

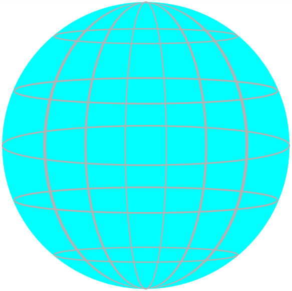

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&= p^{-1}\frac{r^2}{L^2}\left[-p\,dt^2 + p\frac{L^4}{r^2}dr^2 + pL^2 d\Omega_3^2\right]
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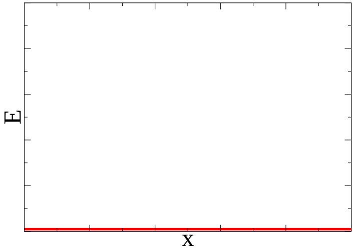

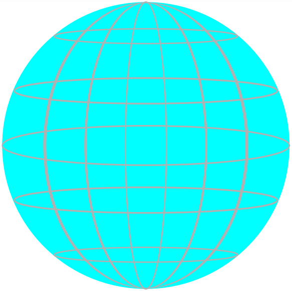

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For SYM on $\mathbb{S}^3$ with arbitrary radius R, $E_{\mathrm{CFT}}R = E_{\mathrm{AdS}}L$.

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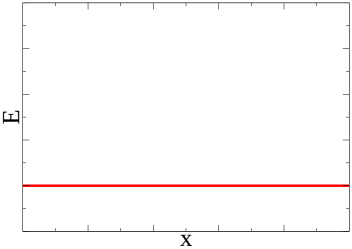

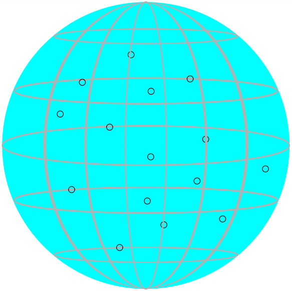

Gas of gravitons in AdS.

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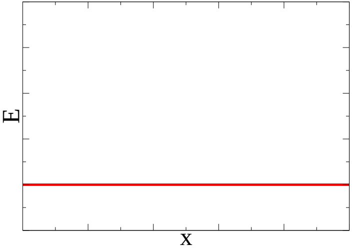

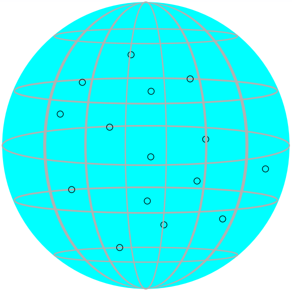

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$$S = \left[\frac{(d+1)^{d+1} d! \omega_d}{(2\pi d)^d} (s\zeta(d+1) + s^* \zeta^*(d+1)) V E^d \right]^{\frac{1}{d+1}}$$

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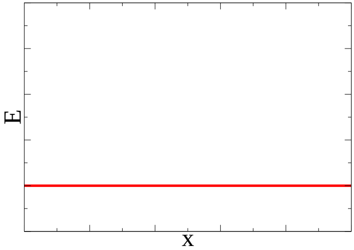

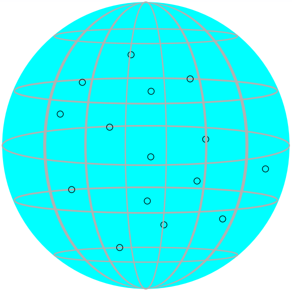

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Use $s = s^* = 128$ and $d = 9$.

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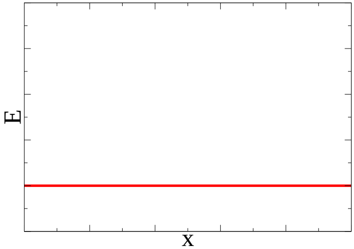

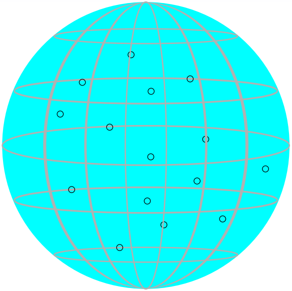

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V has $\omega_3 L^3$ from the $\mathbb{S}^3$ and a piece like L^5 for AdS.

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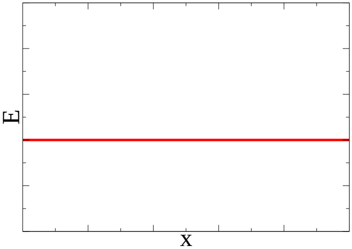
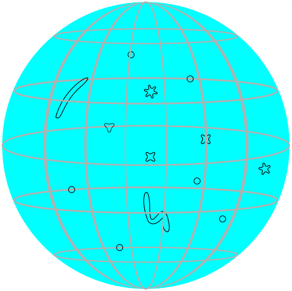
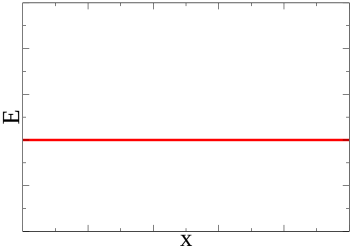
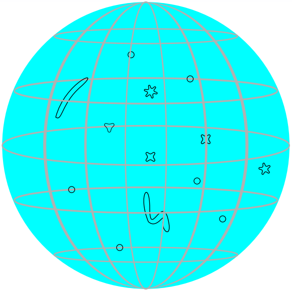




Worldsheets of arbitrarily massive strings.

Gauge theory with one scale





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$$S = \beta_{\mathrm{H}} E$$

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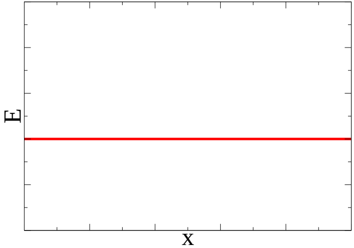

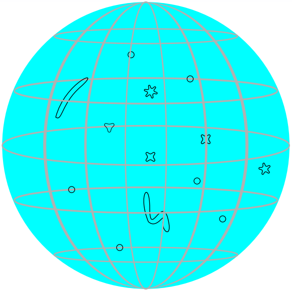

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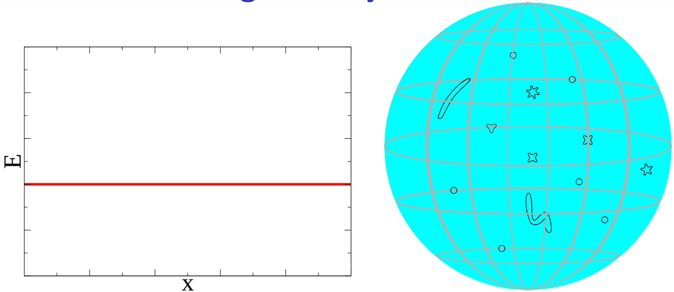

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For Type IIB, $c = \tilde{c} = 12$.

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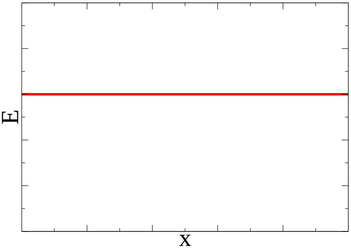

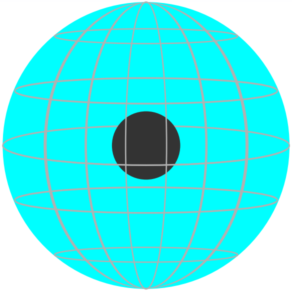

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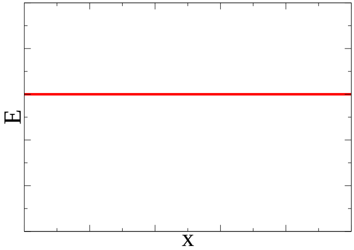

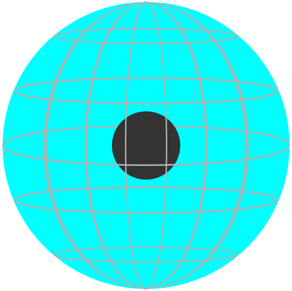

Schwarzschild black hole of mass E:

$$-\left(1-\frac{16\pi GE}{d(d-1)\omega_d r^{d-2}}\right) dt^2 + \left(1-\frac{16\pi GE}{d(d-1)\omega_d r^{d-2}}\right)^{-1} dr^2 \ldots$$

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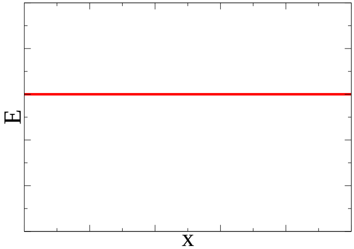

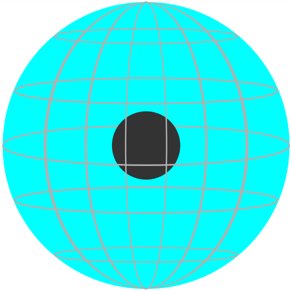

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$$-\left(1-\frac{2\pi G_{10}E}{9\omega_9 r^7}\right)\quad dt^2 \quad +\left(1-\frac{2\pi G_{10}E}{9\omega_9 r^7}\right)^{-1} dr^2 + r^2 d\Omega_8^2$$

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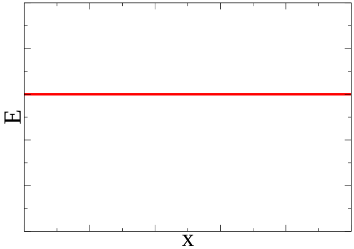

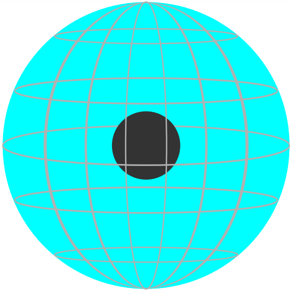

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Horizon radius in terms of energy $\Rightarrow$ Area in terms of horizon radius $\Rightarrow$ $S = \frac{A}{4G_{10}}$.

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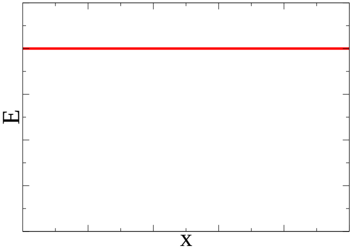

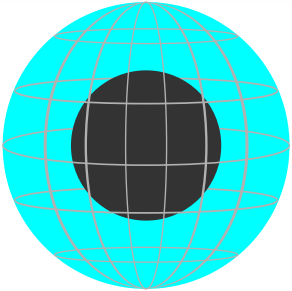

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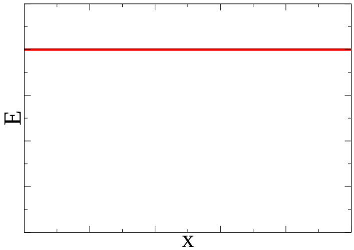

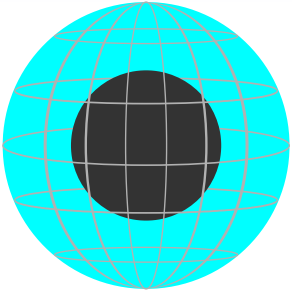

Do the same thing with $S = \frac{A}{4G_5}$ and

$$-\left(1+\frac{r^2}{L^2}-\frac{16\pi GE}{d(d-1)\omega_d r^{d-2}}\right)dt^2+\left(1+\frac{r^2}{L^2}-\frac{16\pi GE}{d(d-1)\omega_d r^{d-2}}\right)^{-1}dr^2$$

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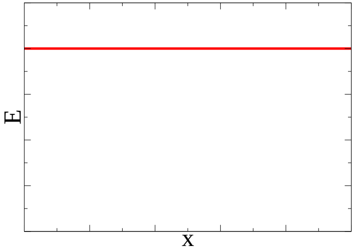

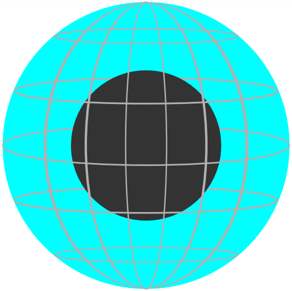

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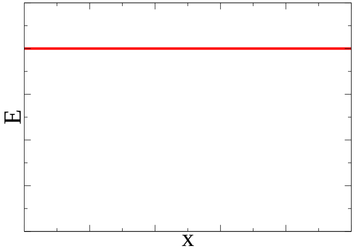
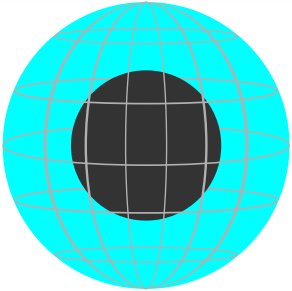
$$-\left(1+\frac{r^2}{L^2}-\frac{4\pi G_5 E}{3\omega_4 r^2}\right)dt^2+\left(1+\frac{r^2}{L^2}-\frac{4\pi G_5 E}{3\omega_4 r^2}\right)dr^2$$

Gauge theory with one scale





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Integrate $\mathbb{S}^5$ out of the Einstein-Hilbert action to get $G_5 = \frac{G_{10}}{6\omega_6 L^5}$.

Gauge theory with one scale

Use $L^4 = 4\pi g_s \alpha'^2 N$, $G_5 L^5 = 8\pi^3 g_s^2 \alpha'^4$ and $\lambda = 4\pi g_s N$ from the correspondence.

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$$S = \begin{cases} 10\left(\frac{2728}{9355}\pi^8\right)^{\frac{1}{10}}\left(\frac{ER}{9}\right)^{\frac{9}{10}} & ER \ll \lambda^{\frac{1}{4}} \\ 2\pi\left(\frac{4}{\lambda}\right)^{\frac{1}{4}} ER & \lambda^{\frac{1}{4}} \ll ER \ll \lambda^{-\frac{7}{4}} N^2 \\ \frac{9}{4}\left(\frac{1890}{N^2}\right)^{\frac{1}{7}}\left(\frac{\pi ER}{9}\right)^{\frac{8}{7}} & \lambda^{-\frac{7}{4}} N^2 \ll ER \ll N^2 \\ \pi\sqrt{N}\left(\frac{4}{3}ER\right)^{\frac{3}{4}} & N^2 \ll ER \end{cases}$$

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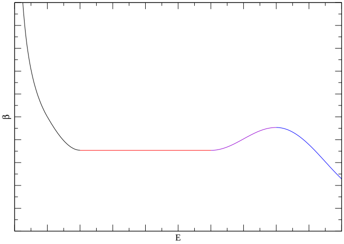

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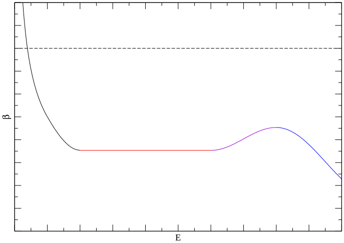

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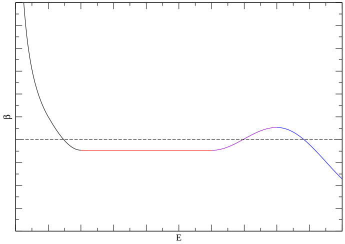
$$S = \begin{cases} 10\left(\frac{2728}{9355}\pi^8\right)^{\frac{1}{10}}\left(\frac{ER}{9}\right)^{\frac{9}{10}} & ER \ll \lambda^{\frac{1}{4}} \\ 2\pi\left(\frac{4}{\lambda}\right)^{\frac{1}{4}} ER & \lambda^{\frac{1}{4}} \ll ER \ll \lambda^{-\frac{7}{4}} N^2 \\ \frac{9}{4}\left(\frac{1890}{N^2}\right)^{\frac{1}{7}}\left(\frac{\pi ER}{9}\right)^{\frac{8}{7}} & \lambda^{-\frac{7}{4}} N^2 \ll ER \ll N^2 \\ \pi\sqrt{N}\left(\frac{4}{3}ER\right)^{\frac{3}{4}} & N^2 \ll ER \end{cases}$$



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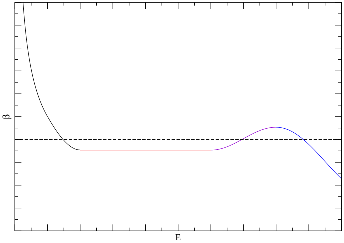

Theory has confinement but it vanishes as $R \to \infty$.

Gauge theory with two scales

$$ds^2 = \frac{L^2}{z^2}\left[-dt^2 + dz^2 + dy^2 + dx_i dx^i\right]$$

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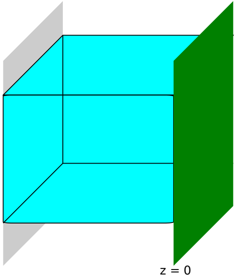

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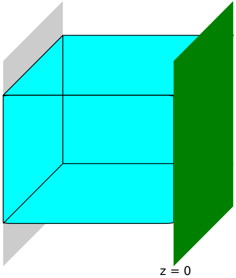

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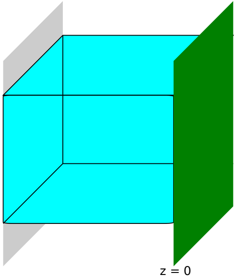

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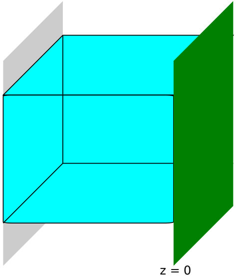

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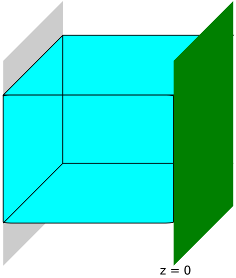

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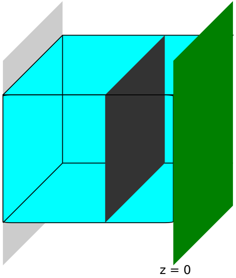

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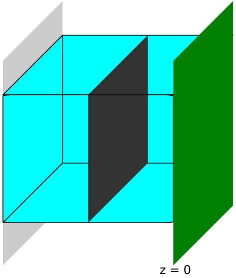

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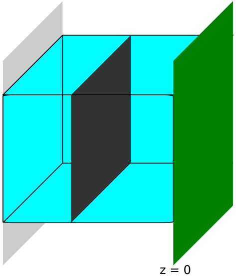

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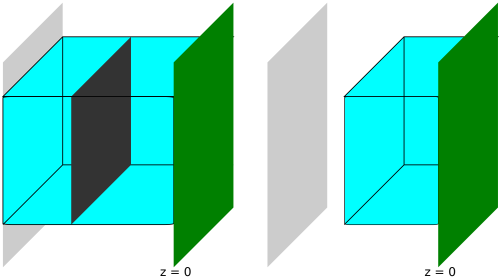

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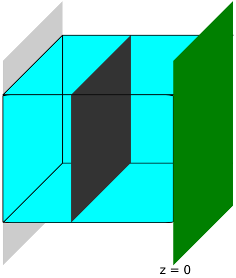
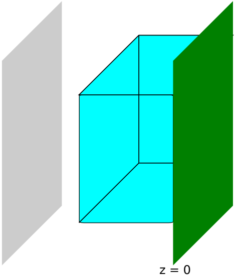
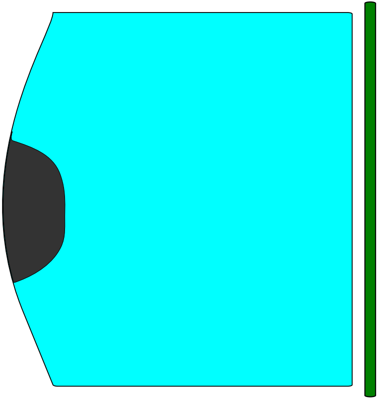




AdS soliton has confined glueballs, AdS black hole has deconfined plasma.

Gauge theory with two scales



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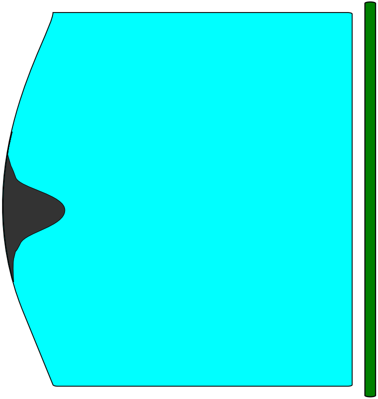

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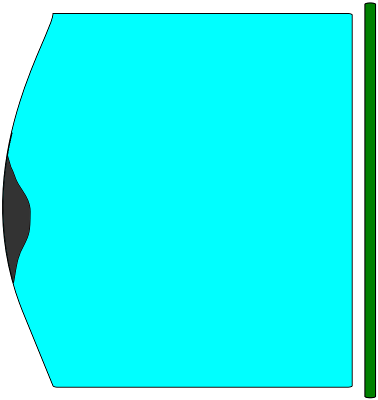

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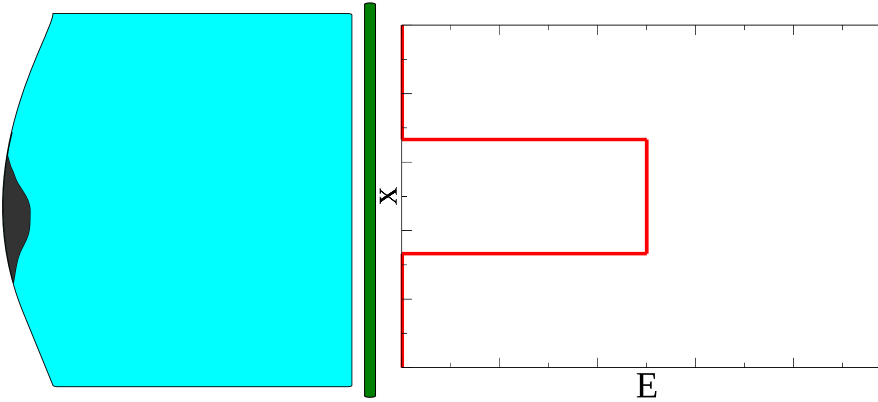

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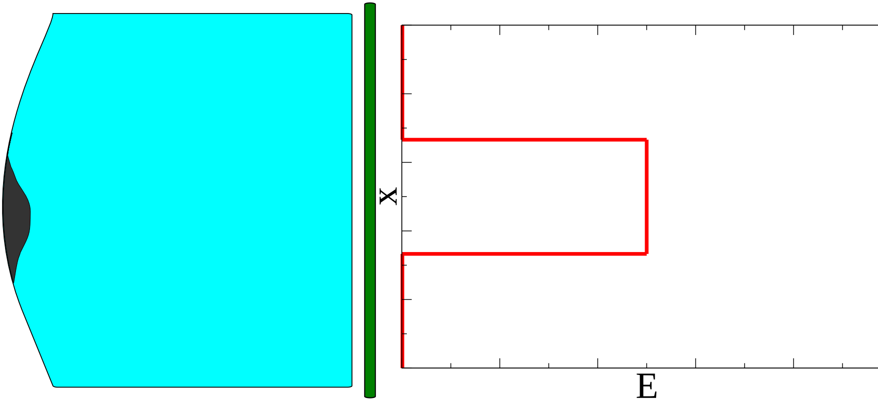
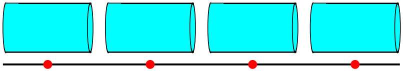
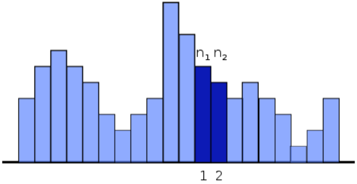


Try to make these objects in a thermodynamic model where there are two scales:

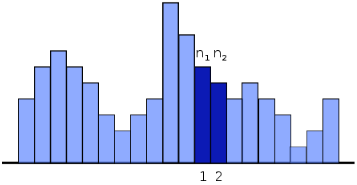


Deriving the model



There are $\rho(n_1)\rho(n_2)$ ways for this to happen. Consider $\rho(n) = Ae^{Bn^\alpha}$.

Deriving the model



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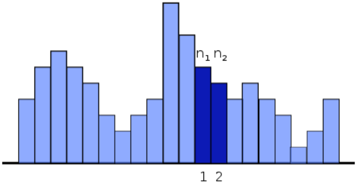
Diffusion	Clustering
$\rho(n_1 - 1)\rho(n_2 + 1) > \rho(n_1)\rho(n_2)$	$\rho(n_1 + 1)\rho(n_2 - 1) > \rho(n_1)\rho(n_2)$
$\alpha < 1$	$\alpha > 1$

Deriving the model



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$\alpha < 1$	$\alpha > 1$

- $\rho(E)$ log-concave
- $S(E)$ concave
- $\beta(E)$ decreasing

Deriving the model

The master equation:

$$\frac{\partial P(\{n_r\})}{\partial t} = \sum_{\{n'_r\}} P(\{n'_r\}) W_{\{n'_r\}\to\{n_r\}} - P(\{n_r\}) W_{\{n_r\}\to\{n'_r\}}$$

$$\frac{\partial \langle n_a \rangle}{\partial t} = \sum_{k\neq 0} \sum_{b} k W_{(n_a,n_b)\to(n_a+k,n_b-k)}$$

Deriving the model

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We want an equilibrium state to be:

$$P\left(\{n_r\}\right) = \frac{1}{Z} \exp(-\beta E) \prod_r \rho(n_r)$$

Deriving the model

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By detailed balance and locality:

$$W_{(n_a,n_b)\to(n_a+k,n_b-k)} = \begin{cases} C\left(\frac{n_a+n_b}{2}\right)\rho(n_a)\rho(n_b) & \text{n.n.} \\ 0 & \text{otherwise} \end{cases}$$

Deriving the model

Continuum limit: $n_a + k$ becomes $E(x) + \epsilon$, $n_b - k$ becomes $E(x + \delta) - \epsilon$. Take the leading term for $\delta, \epsilon \to 0$.

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\begin{aligned}
\frac{\partial E}{\partial t} &= -\delta^2 \epsilon^2 \partial_i \left(C(E) \rho^2(E) \partial_i \frac{\mathrm{d} \log \rho(E)}{\mathrm{d}E} \right) \\
&= -\delta^2 \epsilon^2 \partial_i \left(C(E) \rho^2(E) \partial_i \beta(E) \right)
\end{aligned}
$$

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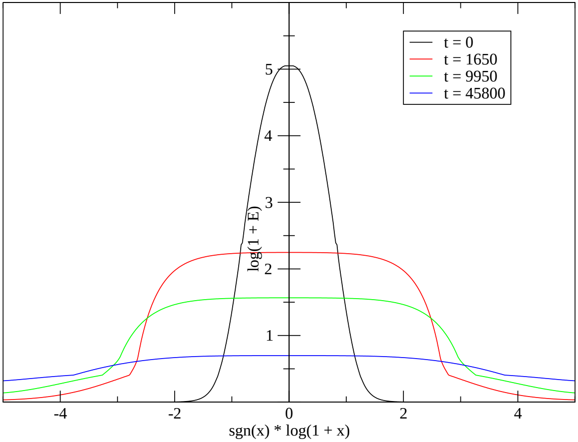
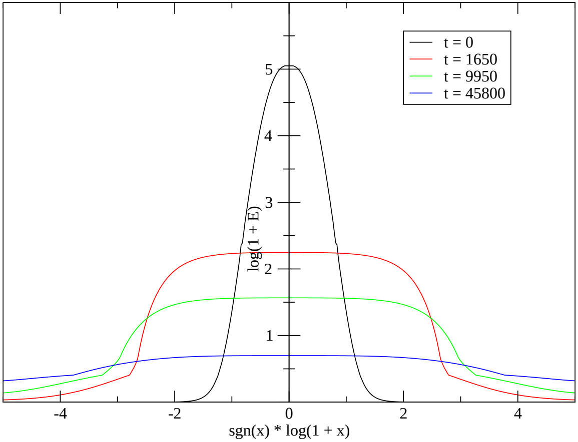
For small fluctuations this PDE is:

- Heat equation for $\alpha < 1$
- Reverse heat equation for $\alpha > 1$
- Static for $\alpha = 1$

Deriving the model

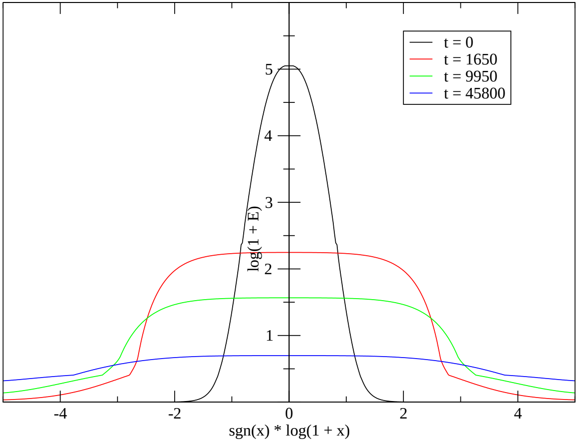


Deriving the model



Use high energies where the model is most effective.

Deriving the model



Use high energies where the model is most effective.
Use Neumann boundary conditions to conserve energy.

Nonlinear diffusion

$$\frac{\partial E}{\partial t} = -\partial_i \left(C(E)\rho^2(E)\partial_i \beta(E) \right)$$

Nonlinear diffusion

$$\begin{aligned}\frac{\partial E}{\partial t} &= -\partial_i \left(C(E)\rho^2(E)\partial_i\beta(E)\right) \\ &= -\Delta\tilde{\beta}(E)\end{aligned}$$

Redefine $\tilde{\beta}'(E) = C(E)\rho^2(E)\beta'(E)$ or just assume $C(E) = \rho^{-2}(E)$.

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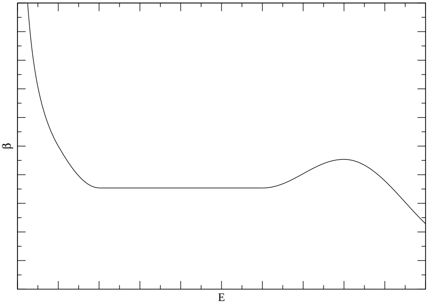

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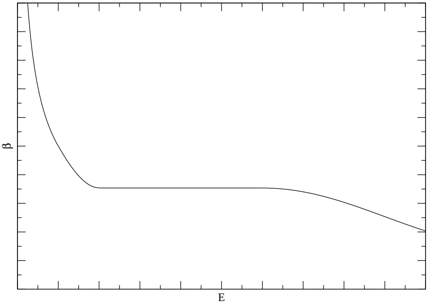
Redefine $\tilde{\beta}'(E) = C(E)\rho^2(E)\beta'(E)$ or just assume $C(E) = \rho^{-2}(E)$. Eliminate the $E^{\frac{1}{7}}$ part of β to make it a decreasing function.



Nonlinear diffusion

$$\begin{aligned}\frac{\partial E}{\partial t} &= -\partial_i \left(C(E)\rho^2(E)\partial_i \beta(E) \right) \\ &= -\Delta\tilde{\beta}(E) \\ &= \Delta\Phi(E)\end{aligned}$$

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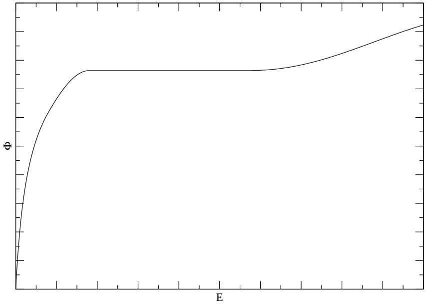

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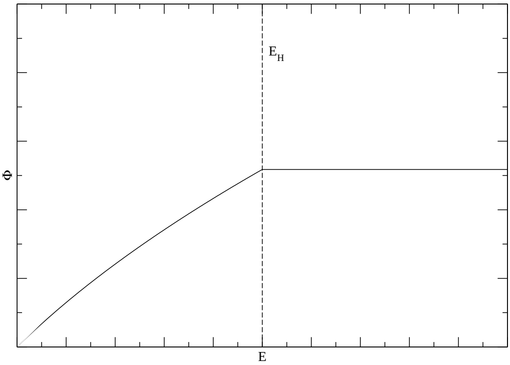
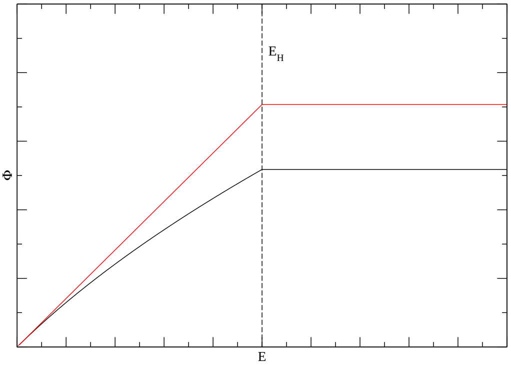


Nonlinear diffusion



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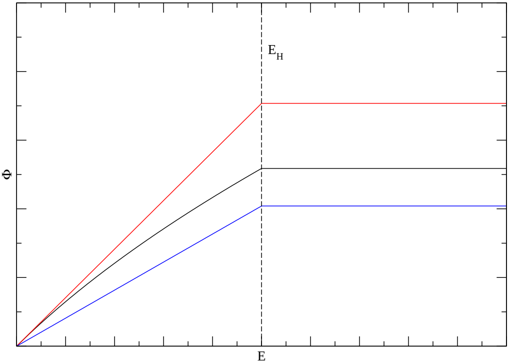
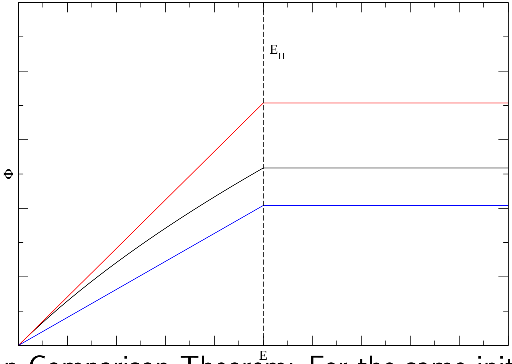


Nonlinear diffusion

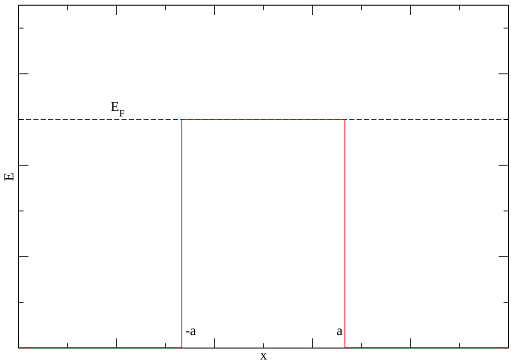


Concentration Comparison Theorem: For the same initial condition, the equations

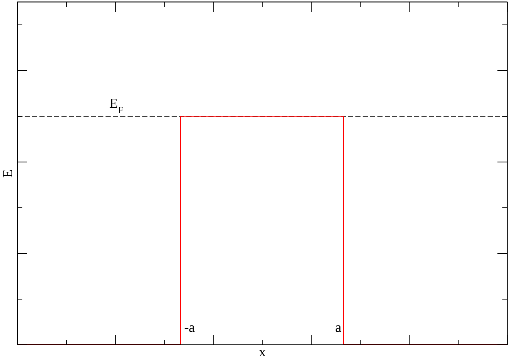
$$\frac{\partial E}{\partial t} = \begin{cases} \Delta \Phi_1(E) \\ \Delta \Phi(E) \\ \Delta \Phi_2(E) \end{cases}$$

satisfy $T_1 < T < T_2$.

Nonlinear diffusion



Nonlinear diffusion



With piecewise linear Φ, this has an exact solution:

$$E(x,t) = \begin{cases} E_{\mathrm{F}} & |x| < a - 2\sqrt{t}I \\ \frac{E_{\mathrm{H}}}{1+\mathrm{erf}(I)}\left(1 + \mathrm{erf}\left(\frac{a-|x|}{2\sqrt{t}}\right)\right) & |x| > a - 2\sqrt{t}I \end{cases}$$

where $\sqrt{\pi} I e^{I^2}\left(1 + \mathrm{erf}(I)\right) = \frac{E_{\mathrm{H}}}{E_{\mathrm{F}} - E_{\mathrm{H}}}$.

Nonlinear diffusion

Use this to find the time for the peak to reach E_{H}.

$$\frac{\pi d}{4(1-\alpha)\beta\left(E_{\min}\right)}\frac{E_{\min}}{E_{\mathrm{H}}^2}\left[aE_{\mathrm{F}}\left(\frac{d-1}{d}\right)^{d-1}\right]^2 < T <$$

$$\frac{\pi d}{4(1-\alpha)\beta\left(E_{\min}\right)}\frac{E_{\min}^{\alpha-1}}{E_{\mathrm{H}}^{\alpha}}\left[aE_{\mathrm{F}}\left(\frac{d-1}{d}\right)^{d-1}\right]^2$$

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If not, $\left(\frac{2-\alpha}{2}\right)^{\frac{2}{\alpha}-\alpha}\beta(E_{\min})^{1-\frac{2}{\alpha}}\frac{E_{\min}^{2-\frac{2}{\alpha}}}{E_{\rm H}^2}e^{1-\frac{2}{\alpha}}$.

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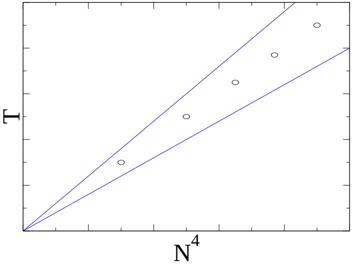
Note that $T = O(N^4) \neq O(N^2)$.

Numerical test

What we want to see:



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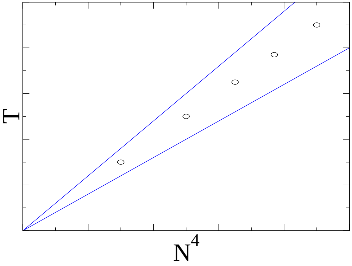
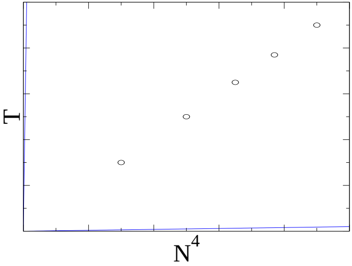


What we actually see:



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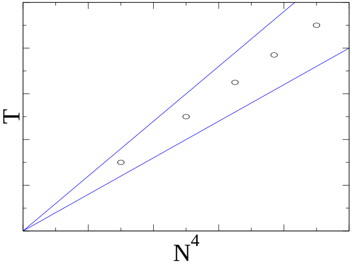

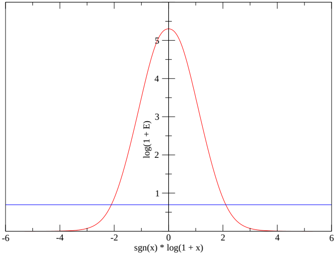

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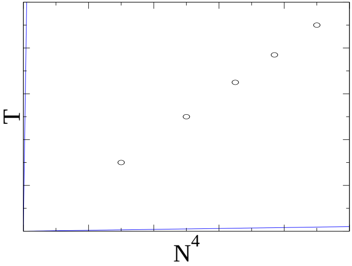

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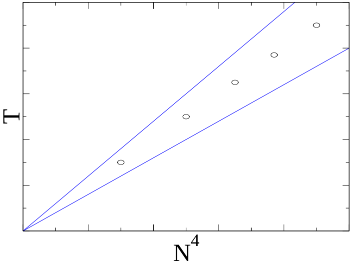

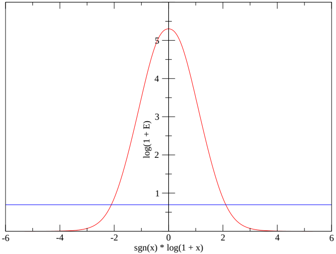

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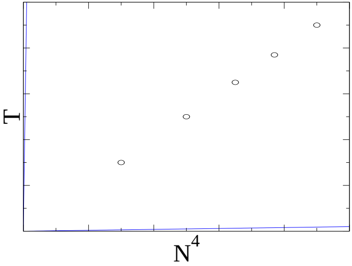

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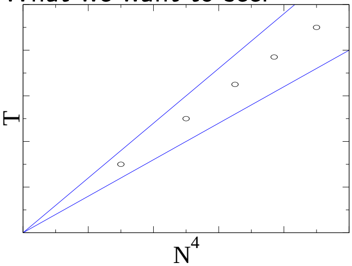

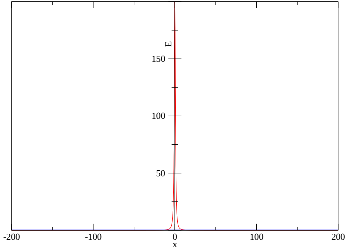

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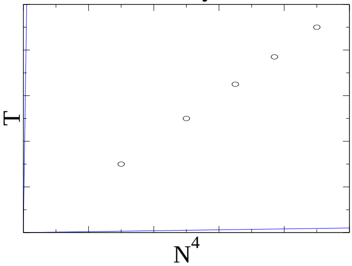

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Numerical test

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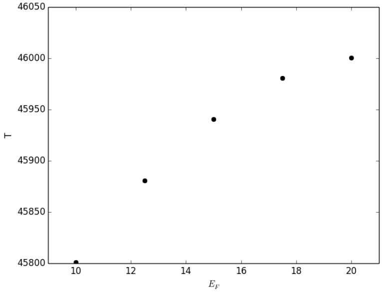
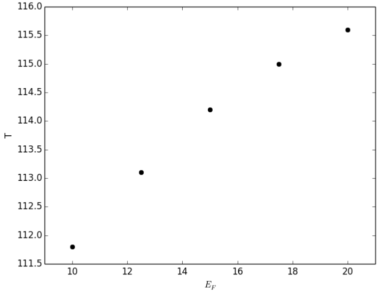


$d = 2$:



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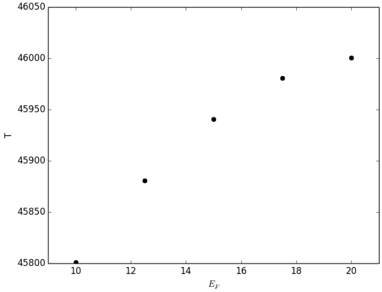

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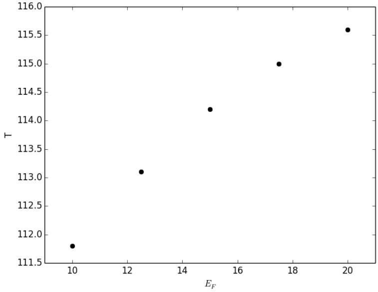

- Bounds are not very constraining.

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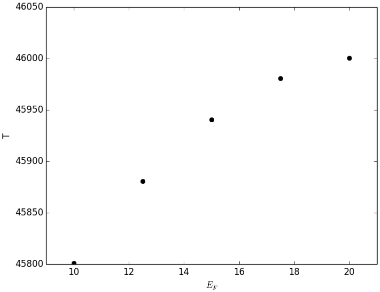
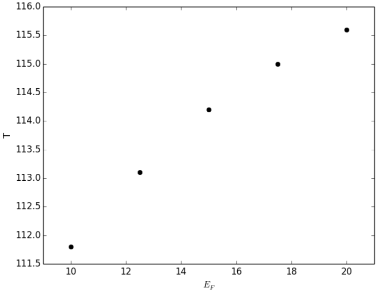


$d = 2$:



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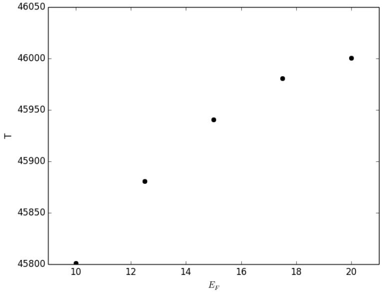

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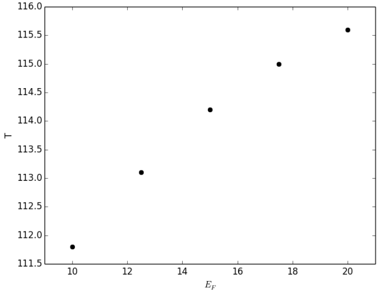

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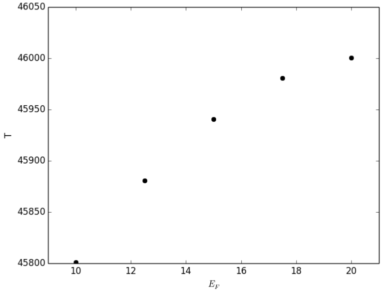

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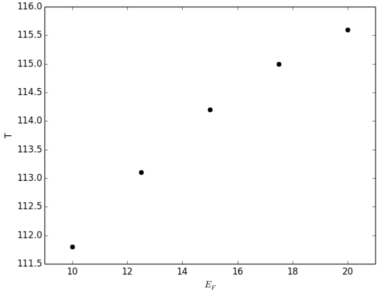

- Bounds are not very constraining.
- This problem is purely mathematical.
- Time in $d = 2$ is much shorter than in $d = 1$.
- There is probably no way around this.

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Barenblatt solution for Dirac delta initial condition:

$$E(x,t) = \left[\frac{\left(\frac{4}{2-\alpha} - 2d\right)t}{|x|^2 + Bt^{\frac{2}{2-d(2-\alpha)}}}\right]^{\frac{1}{2-\alpha}}$$

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Only well defined if $\frac{4}{2-\alpha} - 2d > 0$. Therefore $\alpha > 2 - \frac{2}{d}$ and we can only have $d = 1$ in our case!

Changing the model

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By analogy,

$$W\left(\begin{bmatrix} E(x) & E(x+\delta e) \\ P(x) & P(x+\delta e) \end{bmatrix} \to \begin{bmatrix} E(x)+\epsilon & E(x+\delta e)-\epsilon \\ P(x)+\epsilon e' & P(x+\delta e)-\epsilon e' \end{bmatrix}\right) =$$

$$C\left(\frac{E(x)+E(x+\delta e)}{2}; \frac{P(x)+P(x+\delta e)}{2}\right)$$
$$\rho(E(x); P(x))\rho(E(x+\delta e); P(x+\delta e))\delta_{e,e'}$$

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$$\frac{\partial E}{\partial t} = 0$$
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$$-\frac{\epsilon^3 \delta}{d+2} \partial_l \left[C\rho^2 \left(\frac{\partial^2 \log \rho}{\partial E^2} \delta_{jk} + \frac{\partial^2 \log \rho}{\partial P_j \partial P_k} \right) \right] (\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

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$$
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Numerics are difficult because of expressions for $\rho(E;P)$.

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There is no way $\frac{\partial E}{\partial t} = -\epsilon^2\delta^2\partial_i\left(C\rho^2\partial_i\frac{\partial \log\rho}{\partial E}\right)$ will linearize to $\frac{\partial E}{\partial t} = \partial_i P_i$.

Changing the model

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- Thanks to Klaus Larjo, Nima Lashkari, Brian Swingle and Mark Van Raamsdonk... and all of you.