Black Hole Evaporation from a Random Walk

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An Easy Way and a Hard Way

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- Quantum gravitational effects should be studied using the dual field theory.
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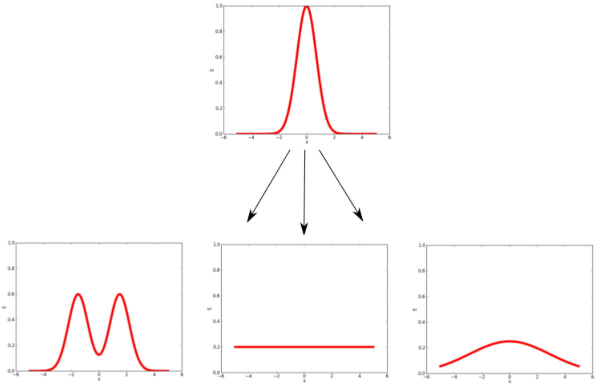

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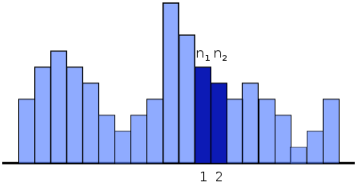

There are $\rho(n_1)\rho(n_2)$ ways for this to happen. Consider $\rho(n) = Ae^{Bn^\alpha}$.

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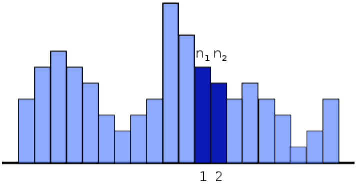

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Diffusion	Clustering
$\rho(n_1 - 1)\rho(n_2 + 1) > \rho(n_1)\rho(n_2)$	$\rho(n_1 + 1)\rho(n_2 - 1) > \rho(n_1)\rho(n_2)$
$\alpha < 1$	$\alpha > 1$

A Master Equation

$$\frac{\partial P(\{n_r\})}{\partial t} = \sum_{\{n'_r\}} P(\{n'_r\}) W(\{n'_r\} \to \{n_r\}) - P(\{n_r\}) W(\{n_r\} \to \{n'_r\})$$

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Assumptions

- $W = 0$ unless $\{n'_r\} = \{\dots n_a + k, n_b - k \dots\}$.
- Nearest neighbour interactions.
- $P(\{n_r\}) = C \prod_r \rho(n_r)$ is an equilibrium state.
- Detailed balance in equilibrium!

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- Nearest neighbour interactions ($b = a + 1$ for instance).
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- Detailed balance in equilibrium!

This leads to

$$W_{(n_a, n_b) \to (n_a + k, n_b - k)} = C \left(\frac{n_a + n_b}{2} \right) \rho(n_a + k) \rho(n_b - k)$$

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Continuum Limit

- $n_a, n_{a+1}, n_{a+2}, \ldots$ become $E(x), E(x+\delta), E(x+2\delta), \ldots$.
- $n_a \pm k$ becomes $E(x) \pm \epsilon$.
- Replace all random variables with expectations.

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$$\begin{aligned}
\frac{\partial E(x)}{\partial t} &= \epsilon C \left(\frac{E(x) + E(x+\delta)}{2} \right) \rho(E(x)+\epsilon) \rho(E(x+\delta)-\epsilon) \\
&\quad + \text{permutations} \\
&= X(\epsilon, \delta) \\
&= \frac{\epsilon^2 \delta^2}{4} \frac{\partial^4 X(0,0)}{\partial \epsilon^2 \partial \delta^2}
\end{aligned}$$

A Nonlinear PDE

$$
\begin{aligned}
\frac{\partial E}{\partial t} &= -\nabla \cdot \left(C(E)\rho^2(E)\nabla \frac{\mathrm{d}\log \rho(E)}{\mathrm{d}E} \right) \\
&= -\nabla \cdot \big(C(E)\rho^2(E)\nabla\beta(E) \big)
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We can mostly consider

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from $C(E) = \rho^{-2}(E)$ or $\tilde{\beta}'(E) = C(E)\rho^2(E)\beta'(E)$.

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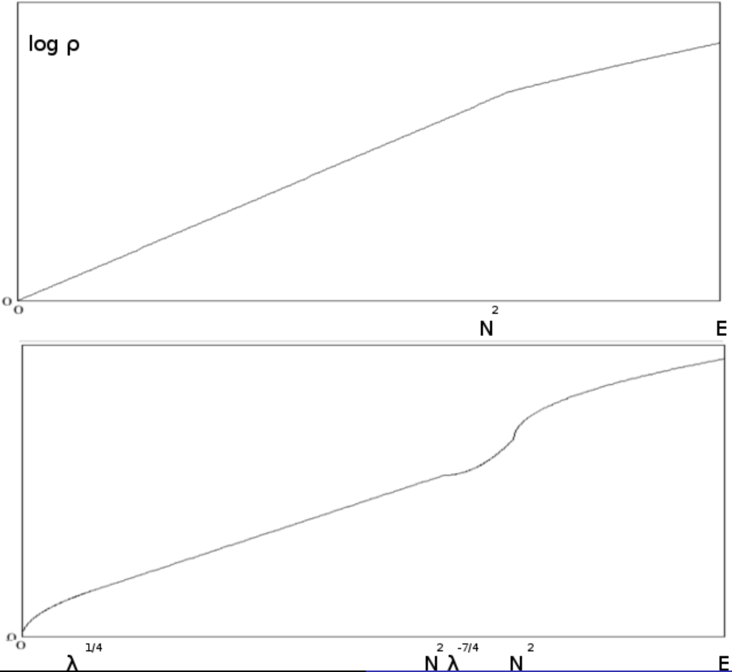
from $C(E) = \rho^{-2}(E)$ or $\tilde{\beta}'(E) = C(E)\rho^2(E)\beta'(E)$. What density of states should we use? $\log\rho(E) \propto E^{\frac{d}{d+1}}$ in a conformal field theory.



log ρ
0
0
2
N
E
0
0
1/4
λ
2 -7/4
N λ
2
N
E

A Nonlinear PDE

$$\frac{\partial E}{\partial t} = -\nabla^2 \beta(E) = \nabla^2 \Phi(E)$$

The density of states appearing here is $\rho(E) = Ae^{BE^\alpha}$.

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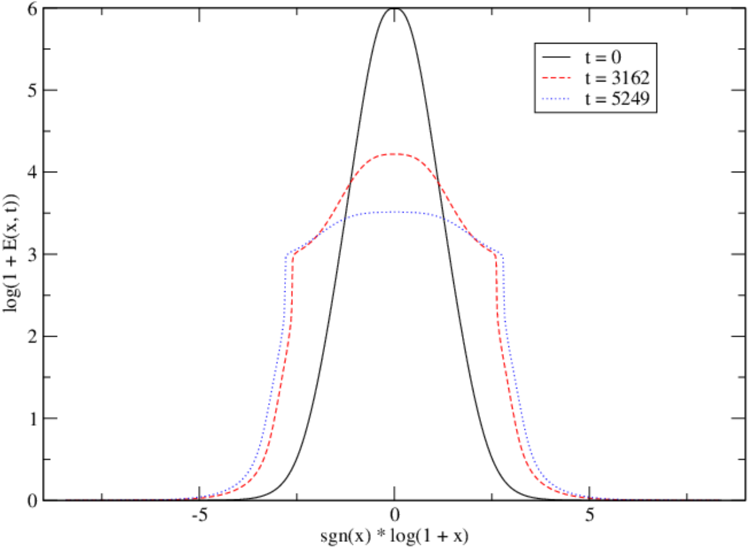
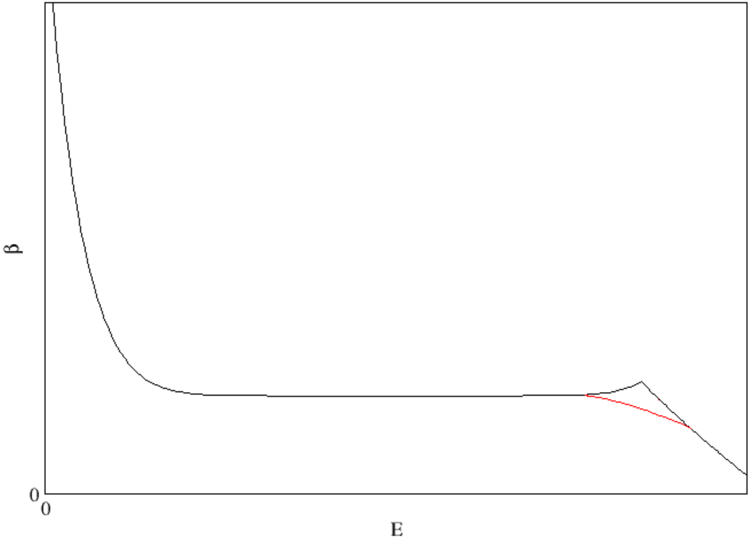
- $\alpha < 1$... energy diffuses like the heat equation.
- $\alpha > 1$... energy clusters like the reverse heat equation.
- $\alpha = 1$... dynamics are frozen in the Hagedorn phase!


t = 0
t = 3162
t = 5249
log(1 + E(x, t))
sgn(x) * log(1 + x)
0
1
2
3
4
5
6
-5
0
5


β
0
0
E

A Nonlinear PDE

Basic properties

- Energy conservation
- Spherical symmetry
- The maximum principle
- Steady state when $t \to \infty$

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Bounds on the decay time:

$$\frac{c^{\alpha}}{1-\alpha}\left(\frac{\alpha E}{2E_H^{\alpha}}\right)^2 < T < \frac{c^{2-\alpha}}{1-\alpha}\left(\frac{E}{E_H}\right)^2$$