Fortuitous states for 4d black holes

Connor Behan

Perimeter Institute

2026-05-12

Based on [2512.23603] with L. Pipolo de Gioia

A 50 year-old observation

This black hole is not stable.

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

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1. Describe black holes using D-branes for small enough string coupling [Strominger, Vafa; hep-th/9601029] .
2. Find "microstate geometries" and their associated CFT (usually CFT_2) states instead of black holes [Mathur; hep-th/0205192] .
3. Look for criteria in a CFT Hilbert space which single out black holes [Chang, Lin; 2208.06728] .

The supersymmetric case

Supersymmetric black holes, annihilated by $Q, Q^\dagger$, are extremal. They exist in many bulk theories including gauged supergravity in AdS_5 [Gutowski, Reall; hep-th/0401042] . Black holes annihilated by additional supercharges seem to be rare [Lin, Lunin, Maldacena; hep-th/0409174] .

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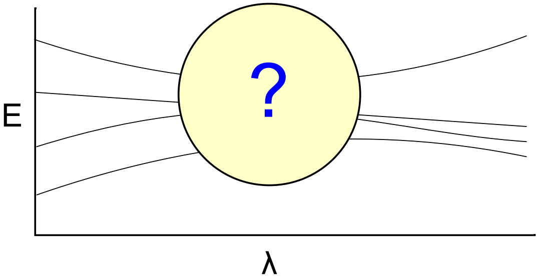

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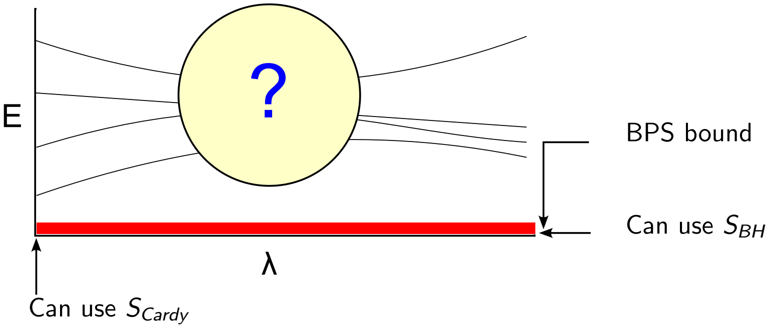

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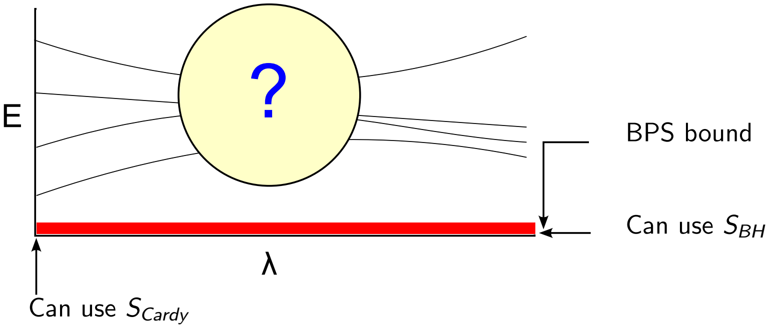

Black holes come from N D0-branes in type IIA $\leftrightarrow$ D1-branes in type IIB. Compactify on $K3 \times S^1$ and use different pictures depending on whether $\lambda = g_s N$ is small or large [Strominger, Vafa; hep-th/9601029] .

Indices for counting states

$$I(q, y) = Tr_{RR}\left[(-1)^F q^{L_0 - c/24} y^{J_0^z}\right]$$

One of many generalizations of [Witten; 82] . For a generic state, $|\psi\rangle$ and $\bar{G}_0 |\psi\rangle$ have the same $\bar{L}_0$ eigenvalue and therefore cancel. A BPS state killed by $\bar{G}_0$ only contributes if there is no state available for it to eat and lift above the BPS bound.

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$$I(q,x,y_2,y_3) = Tr\left[(-1)^F q^{2(E+J_L)} x^{2J_R} y_2^{R_2} y_3^{R_3}\right]$$

When using AdS/CFT to count, there is also a superconformal index for $\mathcal{N}=4$ SYM [Kinney, Maldacena, Minwalla, Raju; hep-th/0510251] . States are labelled by energy E, Lorentz (J_L, J_R) and R-symmetry (R_1, R_2, R_3) which are related by the BPS condition. The action of Q raises E, lowers J_L and changes F so the same principle applies.

$$Q|\psi_1\rangle = 0, \quad Q|\psi_2\rangle = 0 \quad \to \quad Q|\psi_1\rangle \sim g|\psi_2\rangle$$

The holographic approach

The main benefit is the ability to go beyond counting. Consider $U(N)$ or $SU(N)$ gauge theory with maximal supersymmetry.

Operator	Interpretation
$Tr(\Phi^{i_1j_1}\Phi^{i_2j_2})$, $Tr(\Phi^{ij}\bar{\Psi}^k_{\dot{\alpha}})$	5d gravitons
$Tr(\Phi^{i_1j_1}\dots\Phi^{i_pj_p})$	10d graviton
$Tr(\Phi^{i_1j_1}\Phi^{i_2j_2}\Phi^{i_3j_3})Tr(\Phi^{ij}\bar{\Psi}^k_{\dot{\alpha}})$	Multi-graviton
$\epsilon_{ijkl}Tr(\Phi^{ij}\Phi^{kl})$	Excited string with $E = O(\lambda^{1/4})$
$det(\Phi)$	Giant graviton with $E = O(N)$
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The existence of black holes and the independence of multi-traces are both finite N effects.

The fortuity mechanism

There are two ways for a state to be closed.

Monotone: $Q|\psi\rangle = 0$
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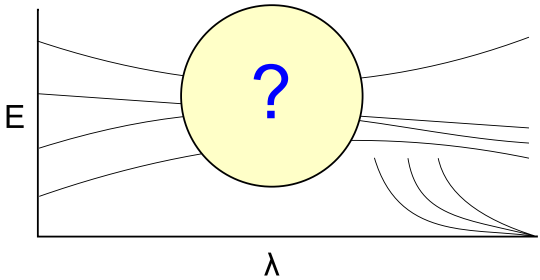
Do fortuitous states exist at all?

- **No**: Early evidence from the index and partition function.
- **Yes**: Later evidence from the index and partition function.

The search for fortuitous states

Initial calculations of the 4d superconformal index gave the graviton gas result $\exp(N^{9/5})$ [Kinney, Maldacena, Minwalla, Raju; hep-th/0510251] . Either the $\exp(N^2)$ fortuitous states cancel almost perfectly or the BPS bound is re-entered.



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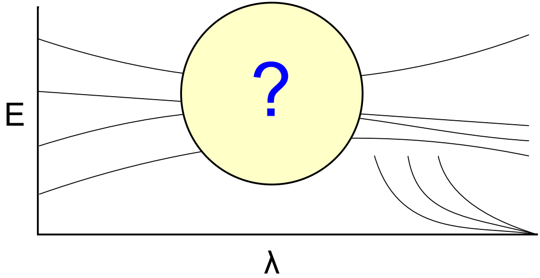
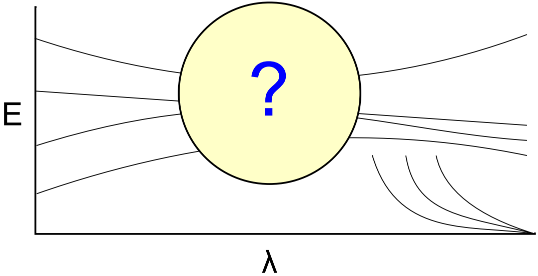


Another approach is to build BPS states level-by-level to get the partition function with unsigned counting. The first several hundred for $SU(2)$ are all monotone [Chang, Yin; 1305.6314] .

How they were found

Including complex saddle points led to the expected $\exp(N^2)$ for $E \sim N^2 \gg 1$ [Cabo-Bizet, Cassani, Martelli, Murthy; 1810.11442] . This shows that enough BPS states at strong coupling come from states that were also BPS at weak coupling.



A more powerful search found one fortuitous state at $O(q^{24})$ with 35075 others [Chang, Lin; 2209.06728] . It was written down compactly soon afterwards [Choi, Kim, Lee, Park; 2209.12696] .

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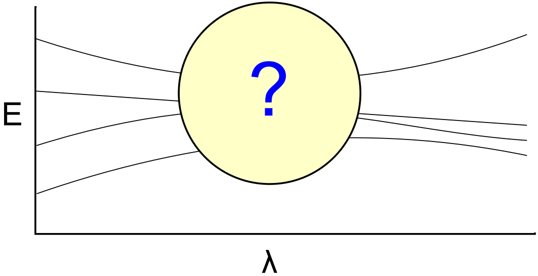

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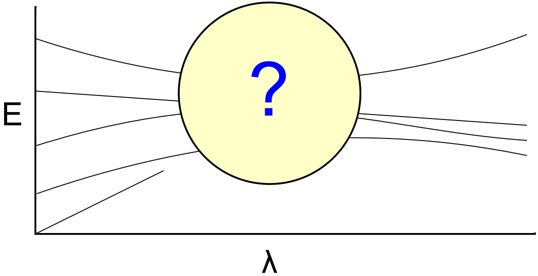

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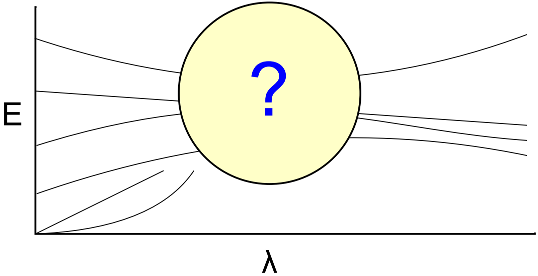

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Loop effects are barely explored

Half-loop

$$\text{SYM with } \lambda = g_{YM}^2 N: \qquad Q = Q^{(0)} + g_{YM} Q^{(1)} + g_{YM}^2 Q^{(2)} + \dots$$
$$\text{ABJM with } \lambda = k^{-1} N: \qquad Q = Q^{(0)} + k^{-1} Q^{(1)} + k^{-2} Q^{(2)} + \dots$$

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Weak coupling spectra with $G = SO(7)$ and $G^L = USp(6)$ have a mismatch [Chang, Lin; 2510.24008] .

$(J_L, J_R, H_1, H_2, H_3)$	$SO(7)$	$USp(6)$
$(\frac{1}{2}, 0, \frac{5}{2}, \frac{5}{2}, \frac{5}{2})$	124	123
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Shown to be a result of the Konishi anomaly (Q not satisfying the Leibniz rule) in [Choi, Lee; 2511.09519] . Compact expression for $Q^{(2)}$ in a convenient scheme was found in [Budzik, Kulp; 2512.07771] .

What to do with $Q^{(1)}$?

SYM: $\{Q^i_\alpha, \bar{Q}_{j\dot{\alpha}}\} = 2\delta^i_j P_{\alpha\dot{\alpha}} \Rightarrow$ use $Q = Q^4_-$.
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Collect the fields killed by $\{Q, Q^\dagger\} \sim E - J - R$ which are BPS letters. These fall into multiplets of the centralizer superalgebra... $\mathfrak{su}(1,2|3)$ for SYM and $\mathfrak{osp}(4|2)$ for ABJM.

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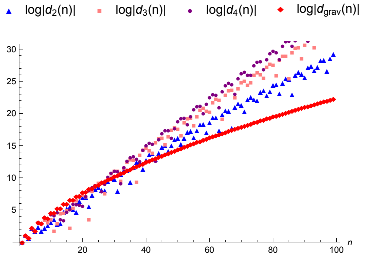
Also do this multi-gravitons where the single-traces are already closed.

This is all we have at large N

[Budzik, Gaiotto, Kulp, Williams, Wu, Yu; 2306.01039] .



[Murthy; 2005.10843]

Results for $\mathcal{N} = 4$ SYM

The BPS letters

$$\phi^n = \Phi^{4n}, \quad \psi_n = -i\Psi_{n+}, \quad \lambda_{\dot\alpha} = \bar\Psi^4_{\dot\alpha}, \quad D_{\dot\alpha} = D_{+\dot\alpha}, \quad f = -iF_{++}$$

transform as

$$\{Q, \lambda_{\dot\alpha}\} = 0, \quad [Q, \phi^n] = 0, \quad \{Q, \psi_m\} = -i\epsilon_{mnp}[\phi^n, \phi^p]$$
$$[Q, f] = i[\phi^n, \psi_n], \quad [Q, D_{\dot\alpha}\zeta\} = -i[\lambda_{\dot\alpha}, \zeta\} + D_{\dot\alpha}[Q, \zeta\}.$$

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Leading (primary) fortuitous operator for $G = SU(2)$ is in $\left(\frac{5}{2}, 0, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right)$ [Chang, Lin; 2209.06728] [Choi, Kim, Lee, Park; 2209.12696] .

$$\mathcal{O}_0 = \epsilon^{m_1 m_2 m_3} Tr(\phi^{m_4}\psi_{m_1}) Tr(\phi^{m_5}\psi_{m_2}) Tr(\psi_{m_3}[\psi_{m_4}, \psi_{m_5}])$$

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$$\begin{aligned}
\mathcal{O}_n &= \epsilon^{m_1 m_2 m_3} \mathrm{Tr}(f^2)^n \mathrm{Tr}(\phi^{m_4}\psi_{m_1}) \mathrm{Tr}(\phi^{m_5}\psi_{m_2}) \mathrm{Tr}(\psi_{m_3}[\psi_{m_4}, \psi_{m_5}]) \\
&+ \frac{n}{2} \epsilon^{m_1 m_2 m_3} \epsilon^{m_4 m_5 m_6} \mathrm{Tr}(f^2)^{n-1} \mathrm{Tr}(f\psi_{m_1}) \mathrm{Tr}(\phi^{m_7}\psi_{m_4}) \mathrm{Tr}(\psi_{m_2}\psi_{m_5}) \mathrm{Tr}(\psi_{m_7}[\psi_{m_3}, \psi_{m_6}]) \\
&+ \frac{n(2n+1)}{6912} \epsilon^{m_1 m_2 m_3} \epsilon^{m_4 m_5 m_6} \epsilon^{m_7 m_8 m_9} \mathrm{Tr}(f^2)^{n-1} \prod_{j=1}^{3} \mathrm{Tr}(\psi_{m_j}[\psi_{m_{j+3}}, \psi_{m_{j+6}}])
\end{aligned}$$

What changes for ABJM theory?

ABJM is a 3d $U(N)_k \times U(N)_{-k}$ theory with $\mathcal{N} = 6$. It is dual to M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ but the $k \to \infty$ 't Hooft limit makes this type IIA on $AdS_4 \times \mathbb{CP}^3$ [Aharony, Bergman, Jafferis, Maldacena; 0806.1218] .

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Spinor / conjugate spinor indices are shown and bar denotes $(\square, \bar{\square})$ while no bar denotes $(\bar{\square}, \square)$.

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Spinor / conjugate spinor indices are shown and bar denotes $(\square, \bar{\square})$ while no bar denotes $(\bar{\square}, \square)$. Project onto $\frac{1}{2}\left(1 + i\Gamma_1\bar{\Gamma}_2\right)$ eigenspaces to get

$$\Phi_A = \begin{bmatrix} \phi_a \\ - \end{bmatrix}, \quad \bar{\Phi}^A = \begin{bmatrix} - \\ \bar{\phi}^a \end{bmatrix}, \quad \Psi^A_+ = \begin{bmatrix} - \\ \psi^a \end{bmatrix}, \quad \bar{\Psi}_{A+} = \begin{bmatrix} \bar{\psi}_a \\ - \end{bmatrix}$$

with the transformations

$$[Q, \phi_a] = 0, \quad \{Q, \psi^a\} = \epsilon^{bc}\phi_b\bar{\phi}^a\phi_c$$
$$[Q, D\zeta\} = \zeta(\bar{\psi}_a\phi_b\epsilon^{ab} - \bar{\phi}^a\psi^b\epsilon_{ab}) + (\psi^a\bar{\phi}^b\epsilon_{ab} - \phi_a\bar{\psi}_b\epsilon^{ab})\zeta + D[Q, \zeta\}$$

and similarly for barred fields.

Fortuitous operators

Primary closed for $N = 2$ [Belin, Singh, Vadala, Zaffaroni; 2512.04146] :

$$\mathcal{O}^a_b = \epsilon^{cd} \left[4\, Tr(\bar{\psi}_c \phi_d \bar{\phi}^a \phi_b) - 3\, Tr(\bar{\psi}_c \phi_d)\, Tr(\bar{\phi}^a \phi_b) \right] - c.c.$$

Primary closed for $N = 2, 3$ [CB, de Gioia; 2512.23603] :

$$\mathcal{O} = \epsilon^{ab} \epsilon_{cd} \epsilon^{ef} [12\, Tr(\bar{\psi}_a \phi_b \bar{\phi}^c \phi_e \bar{\phi}^d \phi_f) - 8\, Tr(\bar{\psi}_a \phi_b \bar{\phi}^c \phi_e)\, Tr(\bar{\phi}^d \phi_f)$$
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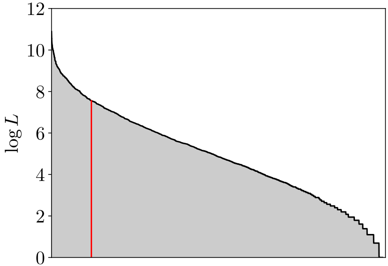
$$+ 3\,Tr(\bar{\psi}_a\phi_b)\,Tr(\bar{\phi}^c\phi_e)\,Tr(\bar{\phi}^d\phi_f)] - c.c.$$

Using $x^{E+J}y^{H_1}$, primaries and descendants are in:

$$\frac{Z_2 - Z_2^{grav}}{\chi} = x^3y^2\chi_1^+\chi_1^- + x^4y^3(1 + \chi_2^+\chi_2^-)$$
$$+ x^5[y^3(\chi_1^+\chi_1^- + \chi_1^+\chi_3^- + \chi_3^+\chi_1^-) + y^4\chi_3^+\chi_3^-] + \dots$$
$$\frac{Z_3 - Z_3^{grav}}{\chi} = x^4y^3(1 + \chi_2^+\chi_2^-) + x^5y^4(2\chi_1^+\chi_1^- + \chi_1^+\chi_3^- + \chi_3^+\chi_1^- + \chi_3^+\chi_3^-) + \dots$$
$$\frac{Z_4 - Z_4^{grav}}{\chi} = x^5y^4(\chi_1^+\chi_1^- + \chi_3^+\chi_3^-) + \dots$$

A possible shortcut to higher levels

It is expensive to compute $Z_N - Z_N^{grav}$ but what about $I_N - I_N^{grav}$? This contains an $O(x^{24})$ term because there is no fortuitous operator with the same quantum numbers as $\{Q, \mathcal{O}_0\}$.



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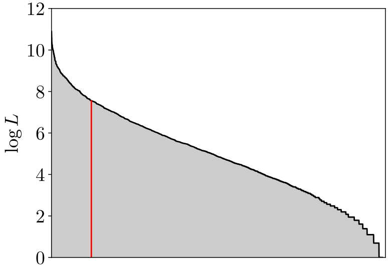

Leaving out $\lambda_{\dot{\alpha}}, D_{\dot{\alpha}}$ gives a consistent truncation of tree-level $\mathcal{N} = 4$ SYM called the BMN sector. Within this sector, one can compute I_2^{grav} exactly [Choi, Kim, Lee, Lee, Park; 2304.10155] .

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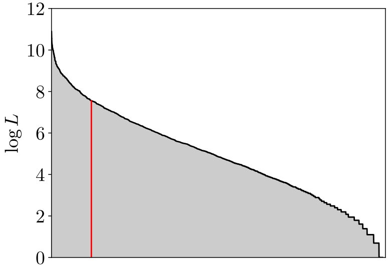

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$$\phi^n \equiv \frac{1}{2}\phi_a\bar{\phi}^b(\sigma^n)_b{}^a, \quad \psi_n \equiv -\frac{1}{2}[\phi_a\bar{\psi}_b(\epsilon\sigma_n)^{ab} + \psi^a\bar{\phi}^b(\sigma_n\epsilon)_{ab}]$$

$$f \equiv \frac{i}{4}[2\psi^a\bar{\psi}_a - \phi_a D\bar{\phi}^a + D\phi_a\bar{\phi}^a]$$

These ABJM bilinears transform like BMN fields [CB, de Gioia; 2512.23603] .

Other developments in fortuity

Black holes should abhor dressing by graviton states. In the BMN tower, $Tr(\phi^i \phi^j)\mathcal{O}_n$ is not a primary because it is exact. Only $Tr(\phi^i f)\mathcal{O}_n$ produces a non-trivial primary [Choi, Kim, Lee, Lee, PArk; 2304.10155] .

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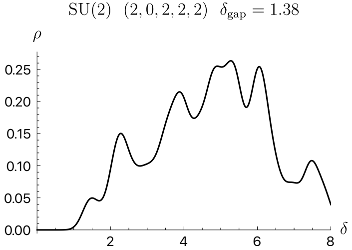
The Hamiltonian $H = \{Q, Q^\dagger\}$ on the classically BPS Hilbert space has $\mathcal{O}_0 \left|0\right\rangle + Q \left|\chi\right\rangle$ as an eigenstate and can be constructed with integrability methods [Budzik, Murali, Vieira; 2306.04693] .



[Chang, Feng, Lin, Tao; 2306.04673]

Future directions

- For small k, monopole operators in ABJM become light and these can be fortuitous as well [Belin, Singh, Vadala, Zaffaronil 2512.04146] .
- It would be very interesting to identify charge sectors with fortuitous operators for a holographic theory with no weak coupling limit like the 6d $\mathcal{N} = (2, 0)$ theory.
- An interesting theory where the BPS spectrum should be explored is 4d $\mathcal{N} = 2$ with $G = SU(2)$ and four fundamental hypers where S-duality acts in an intricate way [Seiberg, Witten; hep-th/9408099] .
- The single-trace cohomology in ABJM should be understood better. Lie algebras and associative algebras become L_∞ and A_∞ algebras. This could lead to a further check of AdS/CFT.

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Thanks for your attention!