Closing all loopholes in quantum devices

Connor Behan

February 25, 2015

The problem

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Bob

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Bob

	Plain	Encrypted
Classical	Trivially insecure	Secure if the pair already knows a secret key
Quantum	Self-destructs upon measurement if the data is randomized	Desired solution

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Key:	010110110001101011001
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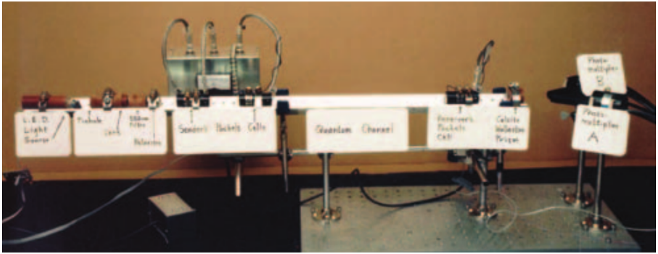

[J Cryptol **5**, 2–38, 1992]

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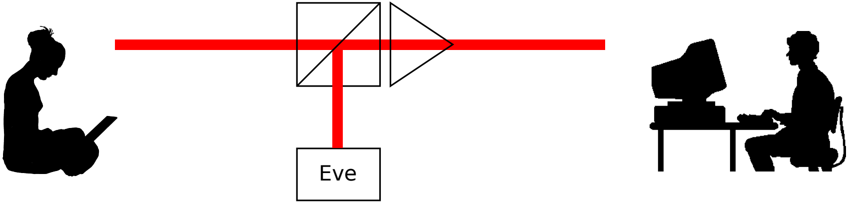

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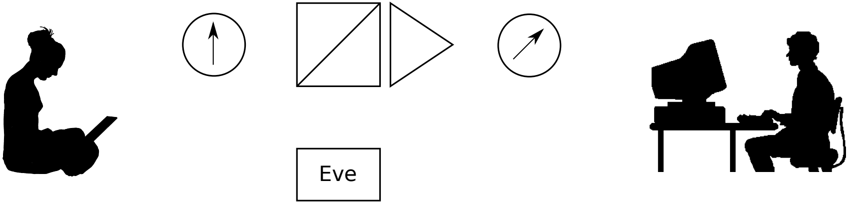

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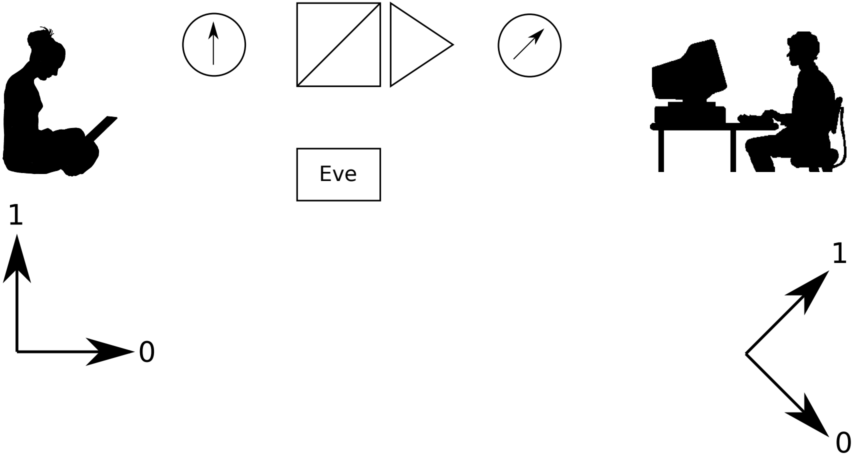

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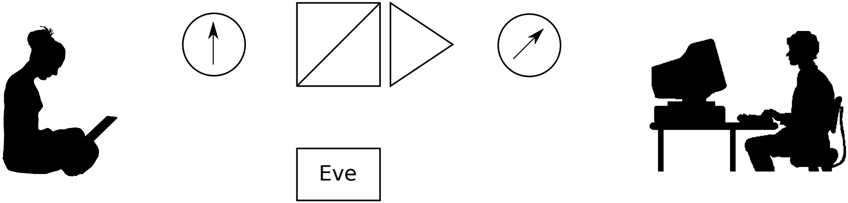
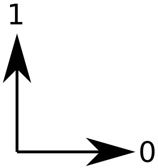
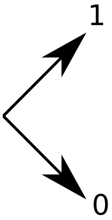




1. Alice encodes with a random basis.
2. Bob measures with a random basis.
3. After all transfers, Alice and Bob publically reveal basis choices.
4. When choices agree, they check agreement on a subset of data.
5. Data not revealed becomes the key.



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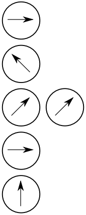

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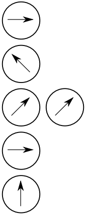
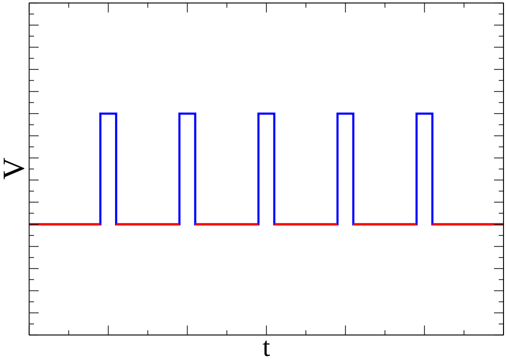


When two photons accidentally arrive, Eve can pass one along, store the other and wait to learn the right basis [PhysRevA **51**, 1863–1869, 1995].

Hacking avalanche photodiodes



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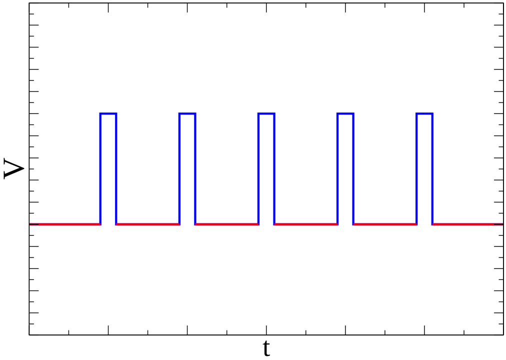

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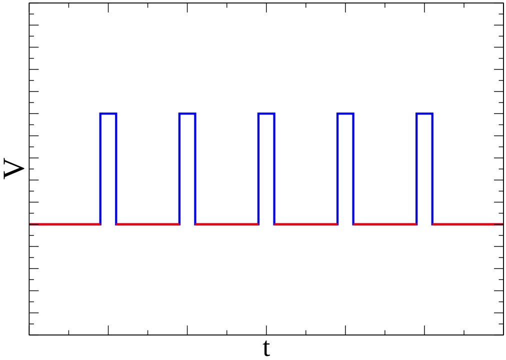

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If Bob's APD is ever in linear mode (*e.g.* the quench after Alice's photon), Eve can use blinding to keep it this way.

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1. Eve chooses bases and measures bits as they come.
2. She retransmits a beam just above P_0 to Bob when his detector is linear.
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Protocols immune to weaknesses in the device are possible with device independent QKD.
Vazirani and Vidick have developed the most robust one to date [PRL **113**, 140501, 2014].

QKD with entanglement

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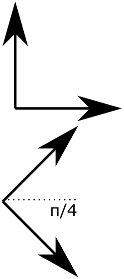

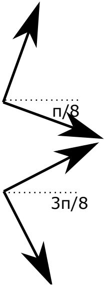

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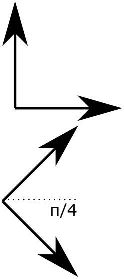
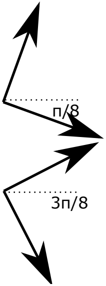


Alice uses input $x_i \in \{0, 1\}$ to decide on basis for output a_i.
Bob uses input $y_i \in \{0, 1\}$ to decide on basis for output b_i.
If outputs saturate the Bell inequality, nothing else can be entangled with the system [PRL **67**, 661–663, 1991].



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For $(x, y) = (0, 0)$, if Alice gets a 0, project onto $\cos\left(\frac{\pi}{8}\right)\langle 0| + \sin\left(\frac{\pi}{8}\right)\langle 1|$.

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$$\left|\left|\rho_{KE} - \rho_{\not K} \otimes \rho_E\right|\right| < \epsilon$$

$$\rho_{\not K} = diag\left(\frac{1}{2^{|K|}}, \ldots, \frac{1}{2^{|K|}}\right)$$

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x	1	2	$\dots$	2^r	$2^r + 1$	$\dots$	2^{r+1}
$f_1(x)$	1	2	$\dots$	2^r	1	$\dots$	2^r
$f_2(x)$	2^r	1	$\dots$	$2^r - 1$	1	$\dots$	2^r
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$f_{2^r}(x)$	2	3	$\dots$	1	1	$\dots$	2^r

Privacy amplification

Leftover Hash Lemma

Supposed we have 2^s *pairwise universal hash-functions that output* r *bit strings. If* $r \leq H_{min}(X) - 2\log_2\left(\frac{1}{\epsilon}\right)$ *and the functions are chosen uniformly,* $(F, F(X))$ *is* ϵ *away from the uniform distribution on* $r + s$ *bits.*

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Proof.

$$
\begin{aligned}
P((F,F(X)) &= (F',F'(X'))) = P(F=F')P(F(X)=F(X')) \\
&= P(F=F')\left[P(X=X') + \frac{1}{2^r}\right] \\
&\le \frac{1}{2^s}\left[\frac{1}{2^q} + \frac{1}{2^r}\right] \\
&= \frac{1}{2^{r+s}}\left[\frac{1}{2^{q-r}} + 1\right] \\
&\le \frac{1}{2^{r+s}}\left[\epsilon^2 + 1\right]
\end{aligned}
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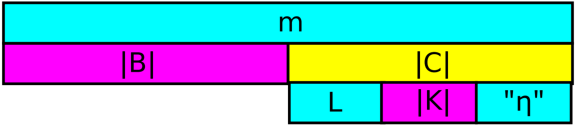
Key length with noise



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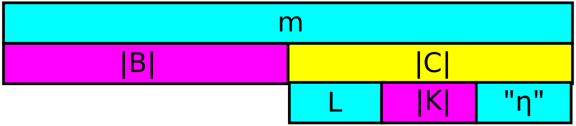

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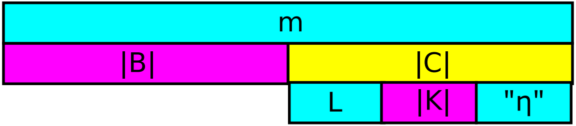

The noise η is defined as the difference between the Bell success probability and $\cos^2\left(\frac{\pi}{8}\right)$. Vazirani and Vidick's bound uses the binary entropy $H(x) = -x\log_2(x) - (1-x)\log_2(1-x)$.

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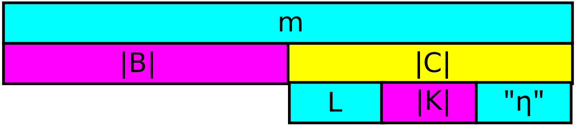

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$$|K| = H^{\epsilon}_{min}(B_C|E) - l - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) \qquad \Big| \ \text{PA}$$

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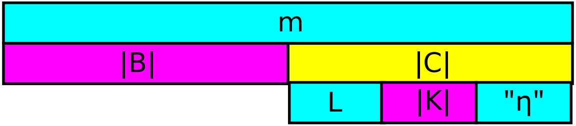

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$$\begin{array}{l|l} |K| = H^{\epsilon}_{min}(B_C|E) - l - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) & \text{PA} \\ |K| \geq H^{\epsilon}_{min}(B_C|E) - H^{\epsilon}_{max}(B_C|A_C) - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) & \text{IR} \end{array}$$

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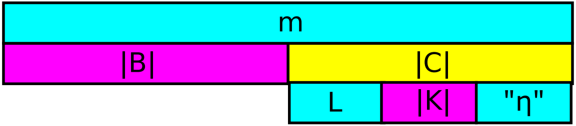

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|K| \geq H^{\epsilon}_{min}(B_C|E) - H\left(\frac{11}{10}\eta\right)|C| - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) & \text{Noise estimate}
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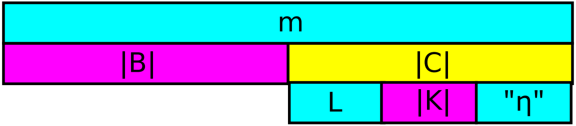

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|K| \geq H^{\epsilon}_{min}(B_C|E) - H\left(\frac{11}{10}\eta\right)|C| - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) & \text{Noise estimate} \\
|K| \geq \kappa(\eta)|C| - H\left(\frac{11}{10}\eta\right)|C| - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) & \text{Rest of the paper}
\end{array}
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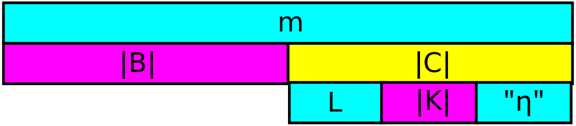
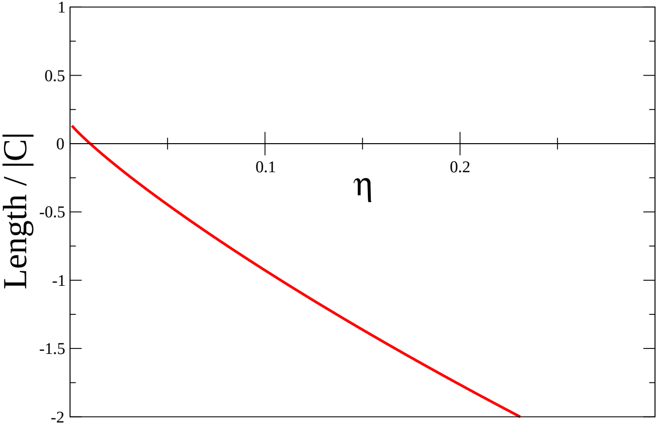
Key length with noise



The noise η is defined as the difference between the Bell success probability and $\cos^2\left(\frac{\pi}{8}\right)$. Vazirani and Vidick's bound uses the binary entropy $H(x) = -x\log_2(x) - (1-x)\log_2(1-x)$.

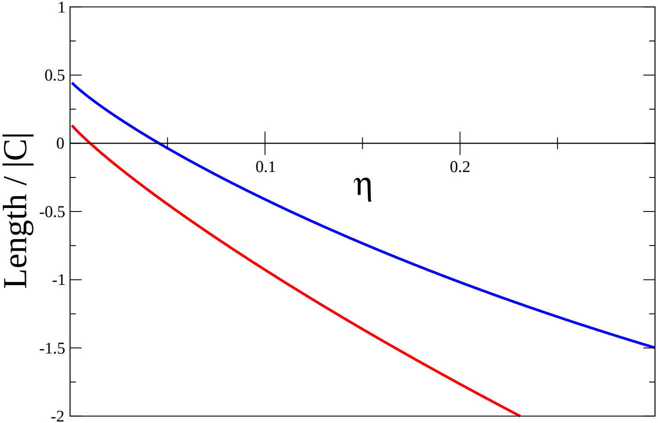
$$
\begin{array}{l|l}
|K| = H^{\epsilon}_{min}(B_C|E) - l - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) & \text{PA} \\
|K| \geq H^{\epsilon}_{min}(B_C|E) - H^{\epsilon}_{max}(B_C|A_C) - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) & \text{IR} \\
|K| \geq H^{\epsilon}_{min}(B_C|E) - H\left(\frac{11}{10}\eta\right)|C| - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) & \text{Noise estimate} \\
|K| \geq \kappa(\eta)|C| - H\left(\frac{11}{10}\eta\right)|C| - O\left(\log_2\left(\frac{1}{\epsilon}\right)\right) & \text{Rest of the paper} \\
|K| \geq \left[\kappa(\eta) - H\left(\frac{11}{10}\eta\right) - O\left(\frac{1}{m}\log_2\left(\frac{1}{\epsilon}\right)\right)\right]|C| & \text{Since } |C| \approx \frac{m}{6}
\end{array}
$$

Key length with noise



$$\kappa(\eta) = \frac{\sqrt{2} - 1}{4 \log 2} - \frac{4}{\log 2}\eta$$

Key length with noise



$$\kappa(\eta) > \frac{\sqrt{2}-1}{4\log 2} - \frac{4}{\log 2}\eta$$

$$\kappa(\eta) = -\frac{11}{3}\log_2\left(\frac{11}{12} + \frac{2}{3}\eta\right)$$

"A pair of entangled photons is like a pair of hippies who are spiritually in tune with one another but not voicing coherent opinions about anything."

—Charles Bennett