Kaons et al: CP violation in the quark sector

Connor Behan

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CPT symmetry

Physics is invariant under CPT.

- C switches particles and antiparticles: $C\phi(x,t) = \phi^*(x,t)$.
- P reflects position: $P\phi(x,t) = \phi(-x,t)$.
- T reflects in time: $T\phi(x,t) = \phi^*(x,-t)$.

Evidence that they are not conserved separately is relatively recent.

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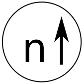

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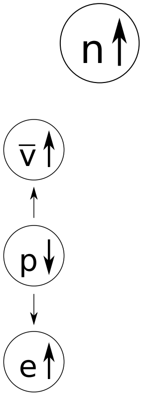

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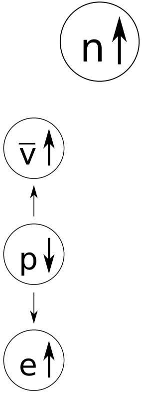

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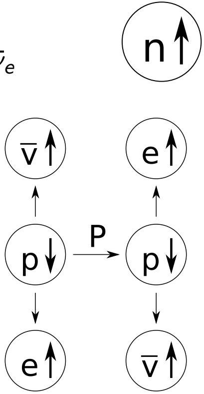

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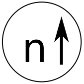

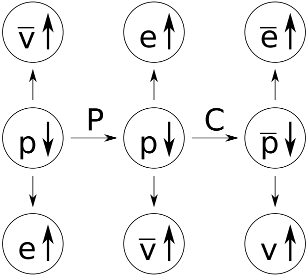

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In 1964, Cronin and Fitch looked at kaons decaying into pions.

$$\left|\pi^0\right\rangle = (|q\uparrow \bar{q}\downarrow\rangle - |q\downarrow \bar{q}\uparrow\rangle) \pm (|\bar{q}\uparrow q\downarrow\rangle - |\bar{q}\downarrow q\uparrow\rangle)$$

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A neutral, CP-odd particle cannot decay into two pions without violating CP!

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Two mass eigenstates of neutral kaons:

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|K_S\rangle &= \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \qquad CP\,|K_S\rangle = |K_S\rangle \\
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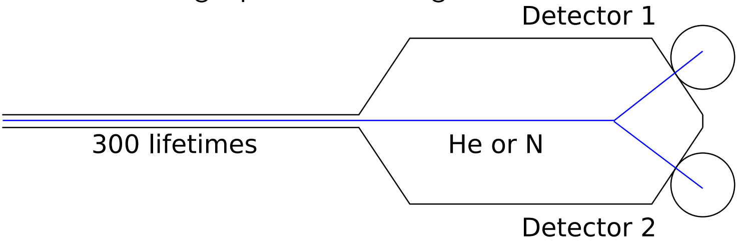

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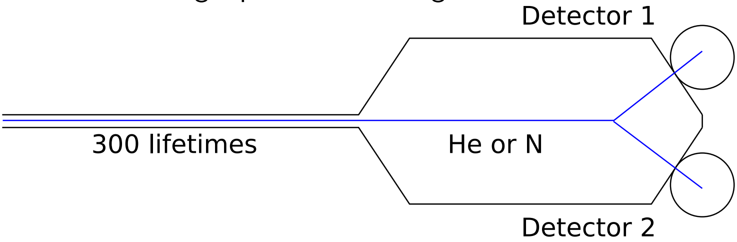

Experiment found $|K_L\rangle$ decaying into two pions!

Standard model explanation

Mass matrices in the standard model are not diagonal.

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\end{aligned}$$

We get a unitary quark mixing matrix called the Cabibbo-Kobayashi-Maskawa matrix: $V_{CKM} = V_{L,u} V^\dagger_{L,d}$.

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&= \frac{g}{\sqrt{2}}\left[\bar{d}_L V^T\gamma^\mu W_\mu^+ u_L + \bar{u}_L\gamma^\mu W_\mu^- V^* d_L\right]
\end{aligned}
$$

This is CP symmetric if and only if V_{CKM} is real!

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$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Measured values are $\sin\theta_{12} = 0.2229 \pm 0.0022$, $\sin\theta_{23} = 0.0412 \pm 0.0020$, $\sin\theta_{13} = 0.0036 \pm 0.0007$, $\delta = 1.02 \pm 0.22$.

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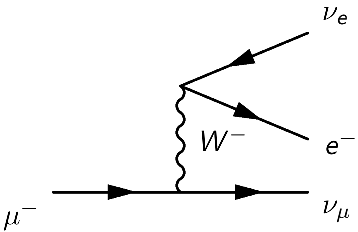
Muon decay:



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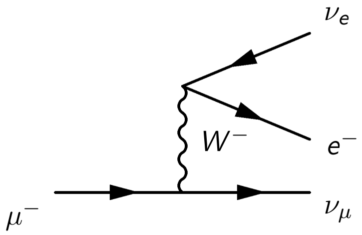
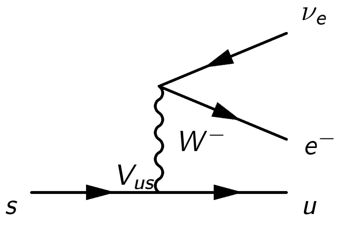


Quark decay:



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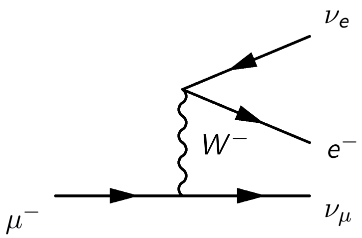

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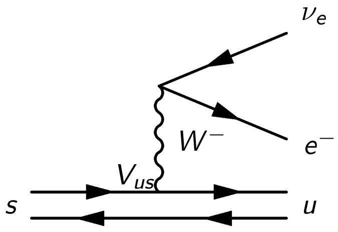

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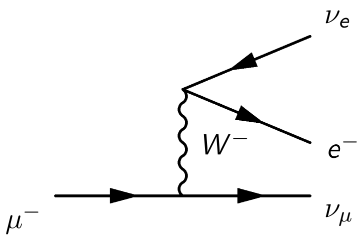

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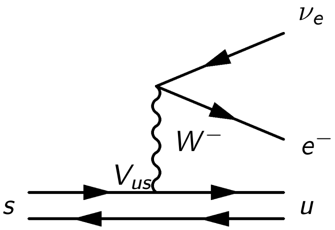

This will measure $|V_{us}|^2$ but not the phase.

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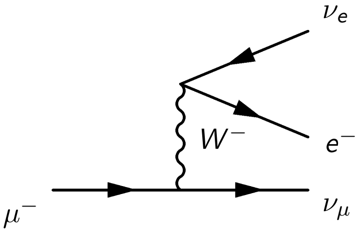

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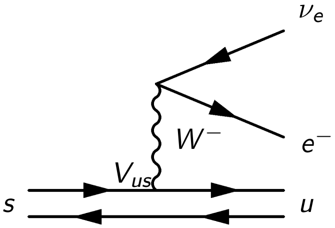

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$$
\begin{aligned}
A &= |A_1|e^{i\theta_1} + |A_2|e^{i\theta_2} \\
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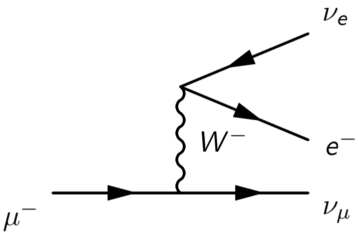

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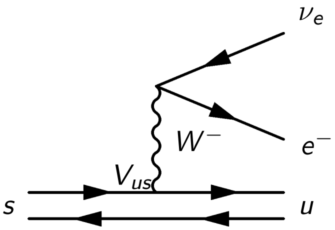

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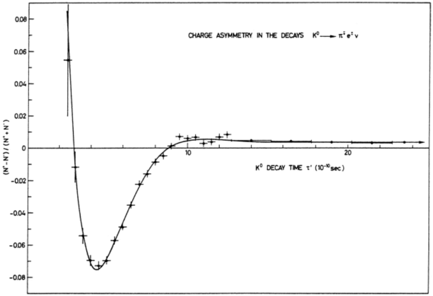

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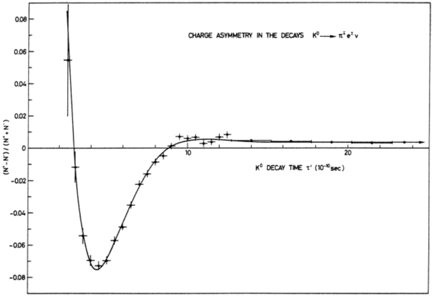

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$$\lim_{t\to\infty} \frac{N_+ - N_-}{N_+ + N_-} = (3.32 \pm 0.06) \cdot 10^{-3}$$

This is a preference for K^0 over $\bar{K}^0$ because $\bar{s}d$ only decays to a positron, $s\bar{d}$ only decays to an electron.

Indirect CP violation

Evolve with a non-Hermitian Hamiltonian:

$$i\frac{d}{dt}\begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix}$$

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$$H = \left[\begin{array}{cc} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{11} - \frac{i}{2}\Gamma_{11} \end{array}\right]$$

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There is indirect CP violation if and only if $\left|\frac{q}{p}\right| \neq 1$.

Indirect CP violation

Working out eigenvectors,

$$\frac{q}{p} = -\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

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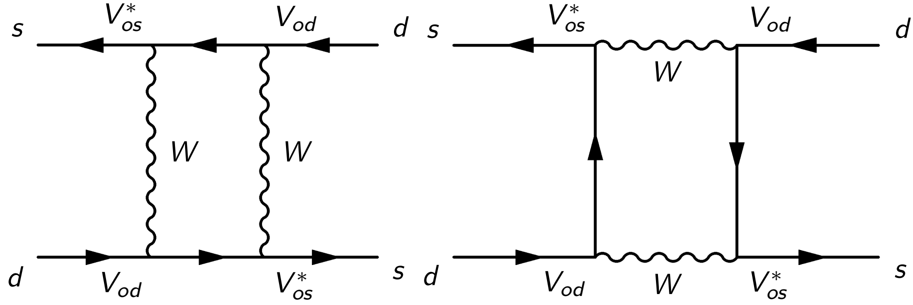
Inami and Lim calculated this from interfering box diagrams.



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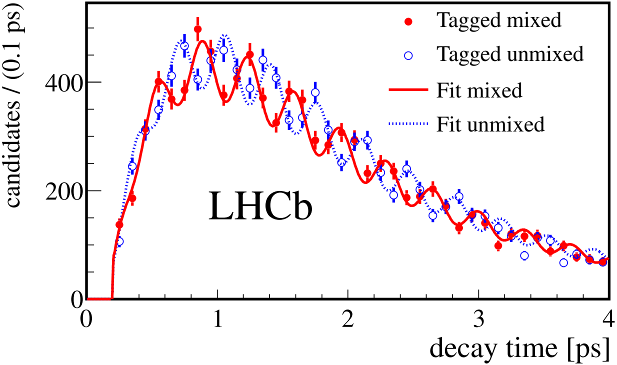

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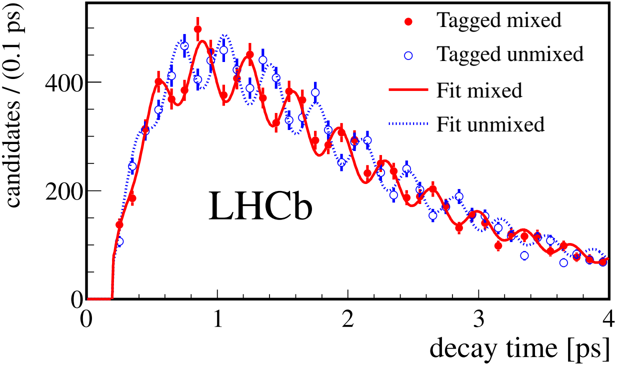

However, they observe

$$\frac{\Gamma(B^0 \to \pi^- K^+) - \Gamma(\bar{B}^0 \to \pi^+ K^-)}{\Gamma(B^0 \to \pi^- K^+) + \Gamma(\bar{B}^0 \to \pi^+ K^-)} = -0.098 \pm 0.012$$

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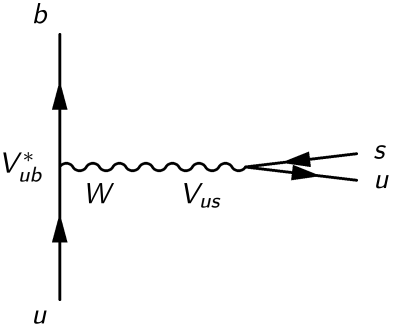

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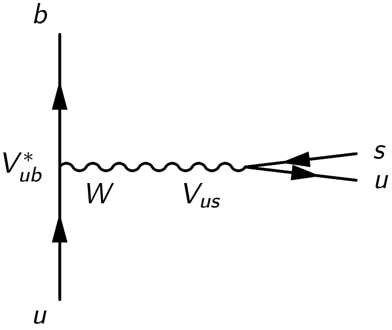

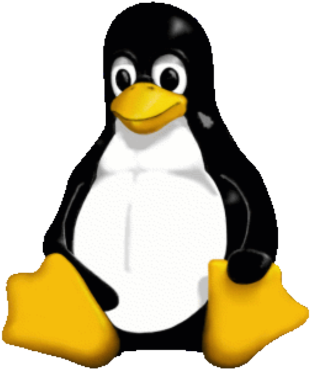

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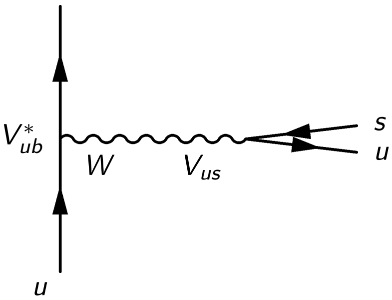

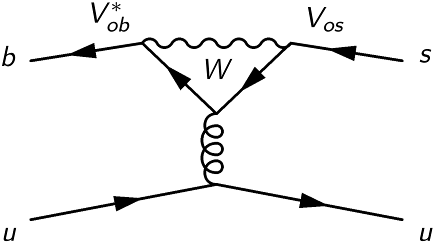

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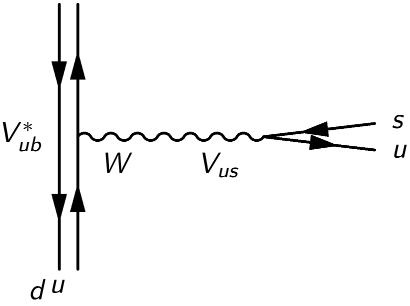

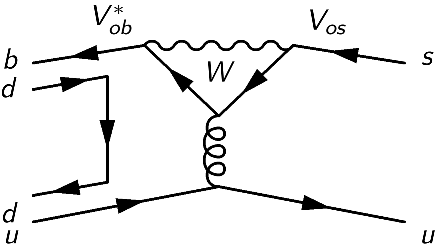

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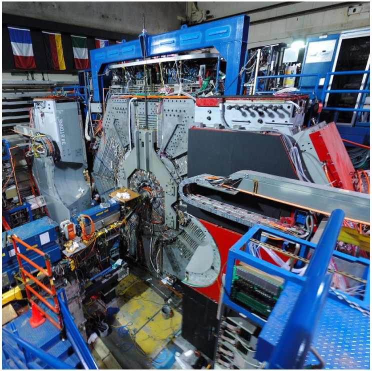

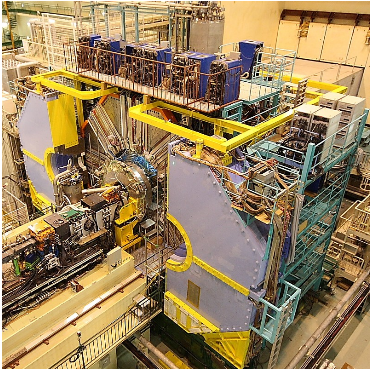

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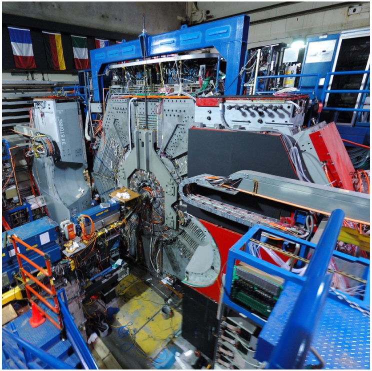

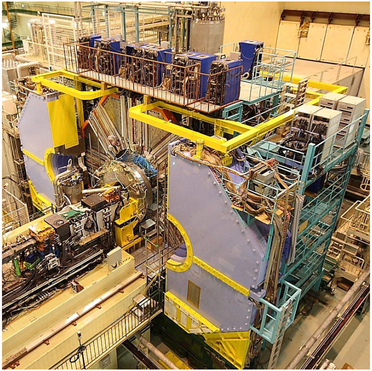

Slight tension in values of $-Im \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}$.

$$B^0 \to K_S^0 \phi : \qquad 0.39 \pm 0.17$$
$$B^0 \to K_S^0 J/\psi : \qquad 0.68 \pm 0.03$$

Plotting the triangle

Since CKM matrix is unitary, 6 entries in $V_{CKM}V_{CKM}^{\dagger}$ must be 0.

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V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* &= 0 \\
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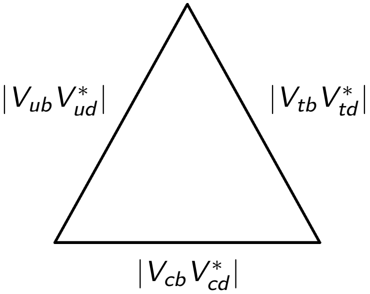
$$



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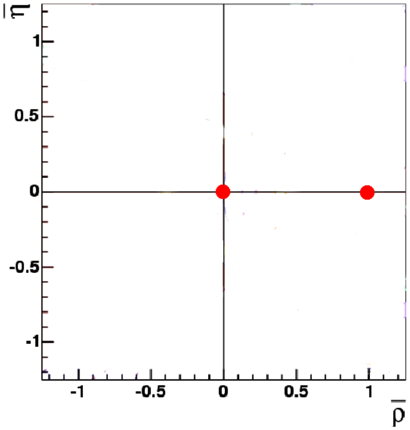

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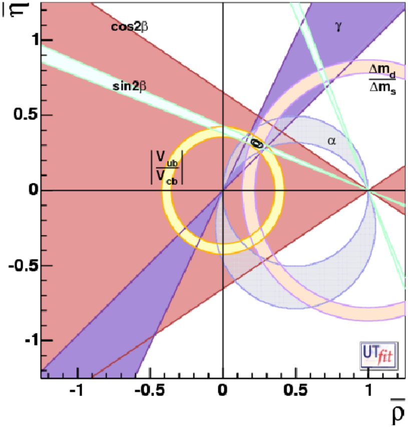

From [arXiv:hep-ph/0509219].

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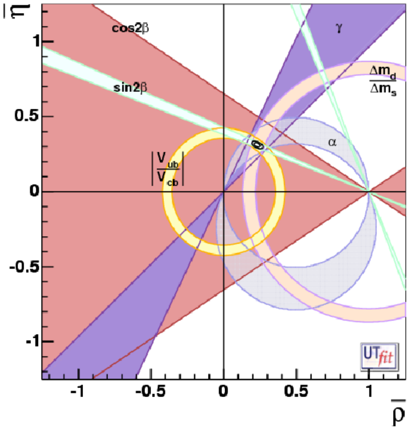

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Stay tuned!

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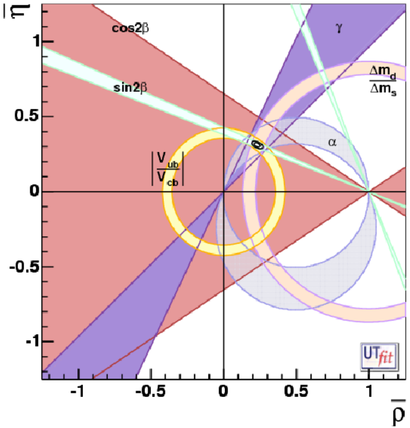

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Thanks Professors McCarthy, Tsybychev.