Large Parameter Limits of Conformal Blocks

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The basics

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$SO(d+1, 1)$ — Euclidean

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$$\left[D, \mathcal{O}^{(\mu)}(0)\right] = i\Delta \mathcal{O}^{(\mu)}(0)$$
$$\left[M_{\mu\nu}M^{\mu\nu}, \mathcal{O}^{(\mu)}(0)\right] = \ell(\ell+d-2)\mathcal{O}^{(\mu)}(0)$$

Primary operators have spin ℓ and scaling dimension Δ.

Correlation functions

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Four or more is a problem: $u \equiv \frac{|x_{12}|^2|x_{34}|^2}{|x_{13}|^2|x_{24}|^2}$ and $v \equiv \frac{|x_{14}|^2|x_{23}|^2}{|x_{13}|^2|x_{24}|^2}$ are conformally invariant.

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$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4) \rangle = \left(\frac{|x_{24}|}{|x_{14}|}\right)^{\Delta_{12}} \left(\frac{|x_{14}|}{|x_{13}|}\right)^{\Delta_{34}} \frac{G(u,v)}{|x_{12}|^{\Delta_1+\Delta_2}|x_{34}|^{\Delta_3+\Delta_4}}$$

Can $G(u,v)$ be anything?

Operator product expansion

A convergent series:

$$\phi_1(x_1)\phi_2(x_2) = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\, C_{(\mu)}\,(x_{12}, \partial_2)\, \mathcal{O}^{(\mu)}(x_2)$$

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Tuning the spectrum will tune the correlation functions. Use the fact that $1 \leftrightarrow 3$ switches u and v. For identical scalars:

$$\begin{aligned} \langle \phi_i(x_1)\phi_i(x_2)\phi_i(x_3)\phi_i(x_4)\rangle &= G(u,v)|x_{12}|^{-2\Delta_i}|x_{34}|^{-2\Delta_i} \\ \langle \phi_i(x_3)\phi_i(x_2)\phi_i(x_1)\phi_i(x_4)\rangle &= G(v,u)|x_{23}|^{-2\Delta_i}|x_{14}|^{-2\Delta_i} \end{aligned}$$

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This yields crossing symmetry:

$$G(u,v) = \left(\frac{v}{u}\right)^{-\Delta_i} G(v,u)$$

Conformal blocks

Four point function OPE: $G(u,v) = 1 + \sum_{\mathcal{O}} \lambda_{12\mathcal{O}} \lambda_{34\mathcal{O}} G_{\mathcal{O}}(u,v)$

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$$\boxed{G_{\mathcal{O}}(u,v) = G^{(d)}_{\Delta,\ell}(u,v;\Delta_{12},\Delta_{34})}$$

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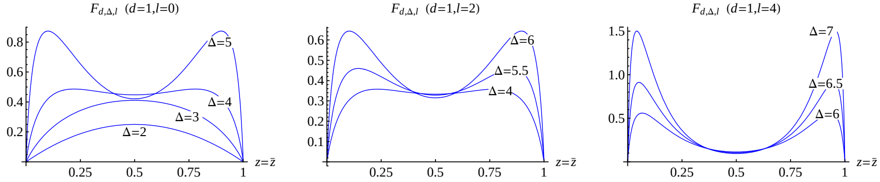
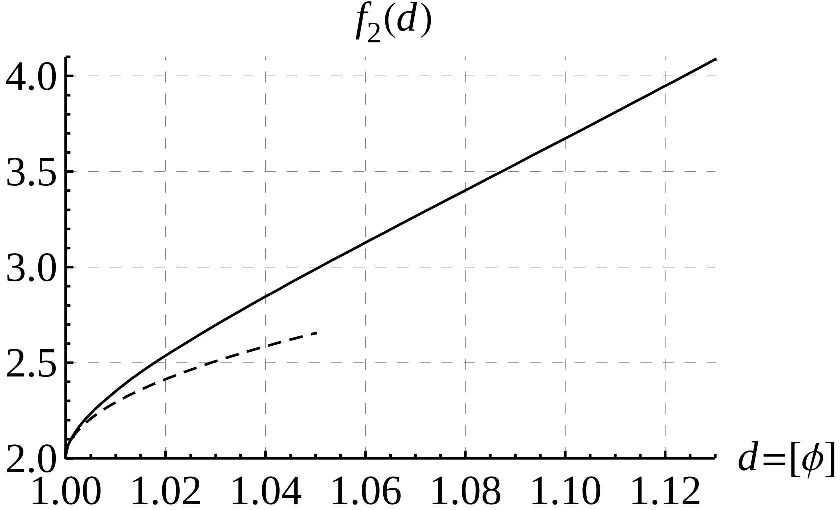
Fix $\Delta_i = 1$, $\ell \in \{0,2,4\}$ and plot a few of these [Rattazzi, Rychkov, Tonni, Vichi, 08]:



The conformal bootstrap



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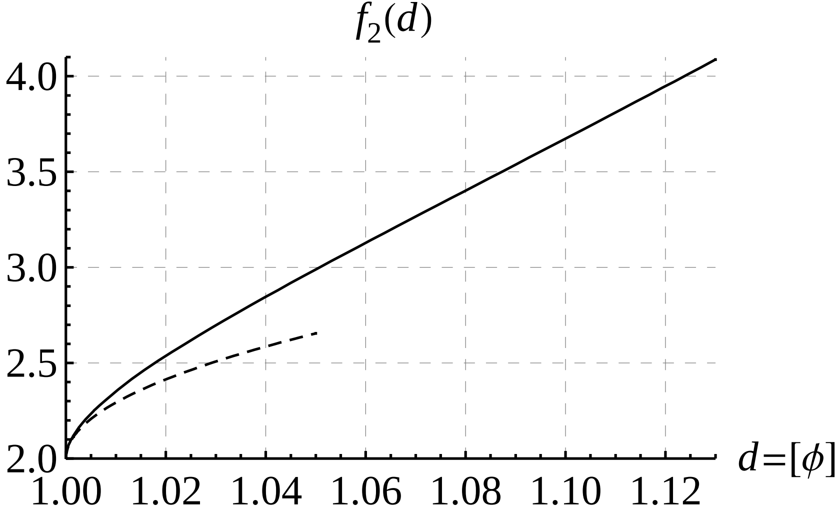

Non-perturbative, theory-independent bound for CFT data [Rattazzi, Rychkov, Tonni, Vichi, 08]!

Hypergeometric functions

Definitions as in [Dolan, Osborn, 00]:

$\lambda_1 = \frac{1}{2}(\Delta + \ell)$	$a = -\frac{1}{2}\Delta_{12}$	$u = xz$
$\lambda_2 = \frac{1}{2}(\Delta - \ell)$	$b = \frac{1}{2}\Delta_{34}$	$v = (1 - x)(1 - z)$

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In $d = 2$:

$$G_{\mathcal{O}}(x,z) = \frac{1}{2} x^{\lambda_1} z^{\lambda_2} {}_2F_1(\lambda_1 + a, \lambda_1 + b; 2\lambda_1; x)\, {}_2F_1(\lambda_2 + a, \lambda_2 + b; 2\lambda_2; z) + (x \leftrightarrow z)$$

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In $d = 4$:

$$G_{\mathcal{O}}(x,z) = \frac{1}{\lambda_1 - \lambda_2 + 1} \frac{1}{x - z} \Big[x^{\lambda_1 + 1} z^{\lambda_2} {}_2F_1(\lambda_1 + a, \lambda_1 + b; 2\lambda_1; x) \; {}_2F_1(\lambda_2 - 1 + a, \lambda_2 - 1 + b; 2\lambda_2 - 2; z) - (x \leftrightarrow z) \Big]$$

The Casimir differential equation

Eigenvalue of $\mathcal{O}(x_2)$ under C_2 is:

$$-2\Lambda_d = -\Delta(\Delta - d) - \ell(\ell + d - 2)$$

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Lift to $(d+1)+1$-dimensional embedding space to handle all terms in the $\phi_1(x_1)\phi_2(x_2)$ OPE:

$$\begin{aligned} C_2 &= \frac{1}{2}\left(L^1_{AB} + L^2_{AB}\right)^2 \\ L^i_{AB} &= P^i_A \frac{\partial}{\partial P^i_B} - P^i_B \frac{\partial}{\partial P^i_A} \end{aligned}$$

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Demand the right eigenvalue for

$$C_2 \left(\frac{|x_{24}|}{|x_{14}|}\right)^{\Delta_{12}} \left(\frac{|x_{14}|}{|x_{13}|}\right)^{\Delta_{34}} \frac{G_{\mathcal{O}}(u,v)}{|x_{12}|^{\Delta_1+\Delta_2}|x_{34}|^{\Delta_3+\Delta_4}}$$

The Casimir differential equation

The result is $D_d G_{\mathcal{O}} = \Lambda_d G_{\mathcal{O}}$ where

$$\begin{aligned}
\Lambda_d &= \lambda_1(\lambda_1 - 1) + \lambda_2(\lambda_2 + 1 - d) \\
D_d &= x^2(1-x)\frac{\partial^2}{\partial x^2} - (a+b)x^2\frac{\partial}{\partial x} - abx + (x \leftrightarrow z) \\
&\quad + (d-2)\frac{xz}{x-z}\left[(1-x)\frac{\partial}{\partial x} - (1-z)\frac{\partial}{\partial z}\right]
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Solution found in $d = 6$ as well. Method generalizes to any even dimension [Dolan, Osborn, 03]. Solutions in odd dimension are not known.

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Consider the change of variables [Fitzpatrick, Kaplan, Poland, 13]:

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For the numerator in $G_{\mathcal{O}}(y_+, y_-) = \frac{H_{\mathcal{O}}(y_+, y_-)}{\sqrt{y_- - y_+}}$,

$$\begin{aligned} H_{\mathcal{O}}(y_+, y_-) &= \sqrt{y_+^{\Delta} y_-^{1-\ell}}\,{}_2F_1\left(\frac{\Delta - 1}{2}, \frac{\Delta}{2}; 1 + \Delta - \frac{d}{2}; y_+\right) \\ &\quad {}_2F_1\left(-\frac{\ell}{2}, \frac{1-\ell}{2}; 2 - \ell - \frac{d}{2}; y_-\right) \end{aligned}$$

The large d limit

The equation satisfied by $H_{\mathcal{O}}$:

$$\Big[2y_+^2(1-y_+)\frac{\partial^2}{\partial y_+^2} - y_+(y_+ + d - 2)\frac{\partial}{\partial y_+} + (y_+ \leftrightarrow y_-) - \frac{y_+ y_-}{2(y_+ - y_-)^2}(y_+ + y_- - 2)\Big] H_{\mathcal{O}} = \left(\Lambda_d - \frac{d-1}{2}\right) H_{\mathcal{O}}$$

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Generalizing to $a, b \neq 0$, this is:

$$2y_+^2(1-y_+)\frac{\partial^2}{\partial y_+^2} - y_+(y_+ + d - 2)\frac{\partial}{\partial y_+} + (y_+ \leftrightarrow y_-)$$
$$- \frac{y_+ y_-}{2(y_+ - y_-)^2}(y_+ + y_- - 2) - 4ab\frac{\sqrt{y_+ y_-} + y_+ y_-}{(\sqrt{y_+} + \sqrt{y_-})^2}$$
$$+2(a+b)\frac{\sqrt{y_+ y_-}}{y_+ - y_-}\left(2y_+(1-y_+)\frac{\partial}{\partial y_+} - (y_+ \leftrightarrow y_-)\right)$$
$$+2(a+b)\frac{\sqrt{y_+ y_-}}{(y_+ - y_-)^2}(y_+^2 - y_+ - y_- + y_-^2)$$

The highly disparate limit

The blocks cease to depend on a and b when the dimension is large. What about a and / or b large?

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$$G_{\mathcal{O}}(x,z) = \frac{1}{a^{\lambda_1+\lambda_2-2b}} \frac{x^b z^b}{(1-x)^{a+b}(1-z)^{a+b}} \left[1 + \frac{1}{2a}\left(\gamma - \frac{1}{x} - \frac{1}{z}\right)(\Lambda_d - b(2b-d)) + \dots\right]$$

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\end{aligned}
$$

Acting with D_d:

$$
\begin{aligned}
D_d G_{\mathcal{O}}(x,z) &= D_d \frac{1}{2a^{\lambda_1+\lambda_2-2b+1}} \frac{x^b z^b}{(1-x)^{a+b}(1-z)^{a+b}} \\
&\quad \left(\gamma - \frac{1}{x} - \frac{1}{z}\right)(\Lambda_d - b(2b-d)) \\
&= \Lambda_d \frac{1}{a^{\lambda_1+\lambda_2-2b}} \frac{x^b z^b}{(1-x)^{a+b}(1-z)^{a+b}} \left(1 + O(a^{-1})\right) \\
&= \Lambda_d G_{\mathcal{O}}(x,z) \left(1 + O(a^{-1})\right)
\end{aligned}
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The highly disparate limit

Main result is

$$G^{(d)}_{\Delta,\ell}(u,v;\Delta_{12},\Delta_{34}) = C^{(d)}_{\Delta,\ell}\frac{v^{\frac{1}{2}\Delta_{12}}}{\left|\frac{1}{2}\Delta_{12}\right|^{\Delta-\Delta_{34}}}\left(\frac{u}{v}\right)^{\frac{1}{2}\Delta_{34}}$$

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To expand the exact solutions, use

$$ {}_2F_1(a,b;c;x) = \frac{1}{B(b,c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tx)^{-a}\mathrm{d}t$$

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They show agreement with

d	$C^{(d)}$
2	$\frac{\Gamma(2\lambda_1)\Gamma(2\lambda_2)}{\Gamma(\lambda_1+b)\Gamma(\lambda_2+b)}$
4	$\frac{\Gamma(2\lambda_1)\Gamma(2\lambda_2-2)}{\Gamma(\lambda_1+b)\Gamma(\lambda_2-1+b)}(\lambda_1+\lambda_2-2)$
6	$\frac{\Gamma(2\lambda_1)\Gamma(2\lambda_2-4)}{\Gamma(\lambda_1+b)\Gamma(\lambda_2-2+b)}(\lambda_1+\lambda_2-3)(\lambda_1+\lambda_2-4)$

The constant

Can we prove this?

$$C^{(d)}_{\lambda_1,\lambda_2} = \frac{\Gamma(2\lambda_1)\Gamma(2\lambda_2+2-d)}{\Gamma\left(\lambda_1+b\right)\Gamma(\lambda_1+\lambda_2+2-d)} \frac{\Gamma\left(\lambda_1+\lambda_2-\frac{d-2}{2}\right)}{\Gamma\left(\lambda_2+b-\frac{d-2}{2}\right)}$$

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For spin zero [Dolan, Osborn, 00] there is a double power series:

$$G^{(d)}_{\lambda,\lambda}(u,v) = \sum_{m,n=0}^{\infty} \frac{(\lambda-a)_m(\lambda-b)_m}{\left(2\lambda-\frac{d-2}{2}\right)_m} \frac{(\lambda+a)_{m+n}(\lambda+b)_{m+n}}{(2\lambda)_{2m+n}} \frac{u^m}{m!}\frac{(1-v)^n}{n!}$$

The constant

Next step is using a recurrence relation from [Dolan, Osborn, 11], $AG_{\lambda_1,\lambda_2} = BG_{\lambda_1-1,\lambda_2+1} + CG_{\lambda_1,\lambda_2+1} + DG_{\lambda_1,\lambda_2+2}$.

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$$
\begin{aligned}
A &= \frac{(\lambda_1+\lambda_2-\varepsilon)(\lambda_1-\lambda_2-1+2\varepsilon)}{\lambda_1-\lambda_2-1+\varepsilon} \\
B &= \frac{\varepsilon(2\lambda_1-1)}{\lambda_1-\lambda_2-1+\varepsilon} \\
D &= -\frac{(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-2\varepsilon)(\lambda_1+\lambda_2+1-2\varepsilon)}{(\lambda_1+\lambda_2-\varepsilon)(\lambda_1+\lambda_2+1-\varepsilon)} \\
&\quad \frac{((\lambda_2+1-\varepsilon)^2-a^2)((\lambda_2+1-\varepsilon)^2-b^2)}{4(\lambda_2+1-\varepsilon)^2(4(\lambda_2+1-\varepsilon)^2-1)} \\
\varepsilon &= \frac{d-2}{2}
\end{aligned}
$$

The constant

$$
\begin{aligned}
2C \quad = \quad & (\lambda_1+\lambda_2-2\varepsilon)\left(\frac{1}{x}+\frac{1}{z}-1\right.\\
&\left.+\frac{ab}{2}\frac{\lambda_1(\lambda_1-1)+(\lambda_2+1)(\lambda_2-2\varepsilon)+2\varepsilon}{\lambda_1(\lambda_1-1)(\lambda_2+1-\varepsilon)(\lambda_2-\varepsilon)}\right)\\
&+\frac{1}{\lambda_1-\lambda_2-1}\left[\frac{x-z}{xz}\left(x^2(1-x)\frac{\partial^2}{\partial x^2}-x^2\frac{\partial}{\partial x}-(x\leftrightarrow z)\right)\right.\\
&\left.+\frac{ab}{2}\frac{(\lambda_1-\lambda_2-1)(\lambda_1-\lambda_2-1+2\varepsilon)(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-2\varepsilon)}{\lambda_1(\lambda_1-1)(\lambda_2+1-\varepsilon)(\lambda_2-\varepsilon)}\right]
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&\quad +\frac{1}{\lambda_1-\lambda_2-1}\left[\frac{x-z}{xz}\left(x^2(1-x)\frac{\partial^2}{\partial x^2}-x^2\frac{\partial}{\partial x}-(x\leftrightarrow z)\right)\right. \\
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\end{aligned}
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We know the spacetime dependence of the asymptotic blocks so it suffices to consider the diagonal limit $x=z$ [Hogervorst, Osborn, Rychkov, 13].

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& +\frac{1}{\lambda_1-\lambda_2-1}\left[\frac{x-z}{xz}\left(x^2(1-x)\frac{\partial^2}{\partial x^2}-x^2\frac{\partial}{\partial x}-(x\leftrightarrow z)\right)\right.\\
& \left.+\frac{ab}{2}\frac{(\lambda_1-\lambda_2-1)(\lambda_1-\lambda_2-1+2\varepsilon)(\lambda_1+\lambda_2)(\lambda_1+\lambda_2-2\varepsilon)}{\lambda_1(\lambda_1-1)(\lambda_2+1-\varepsilon)(\lambda_2-\varepsilon)}\right]
\end{aligned}
$$

We know the spacetime dependence of the asymptotic blocks so it suffices to consider the diagonal limit $x = z$ [Hogervorst, Osborn, Rychkov, 13]. This confirms our guess [B, 14].

Convergence

Does the sum

$$G(u,v) = \sum_{\mathcal{O}} \lambda_{12\mathcal{O}} \lambda_{34\mathcal{O}} G_{\mathcal{O}}(u,v)$$

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No convergence, so using the first few terms ($\Delta \ll |\Delta_{12}|$) is risky.

Spinning case

For non-scalar $\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$, the spinning conformal blocks are only known in $d = 2$ [Osborn, 12].

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- There are five differential operators [Costa, Penedones, Poland, Rychkov, 11] that must be applied in various orders to the non-spinning blocks.
- There is always one combination where this is pure multiplication. Large-a and large-d limits still apply.

Future ideas

- Check the approximation by performing a large-a bootstrap in $d = 2$ or $d = 4$.
- Take large-a or large-d limits of specific spinning conformal blocks and look for patterns.
- Perhaps large-ℓ_{12} limit is possible too.
- Analyze the Casimir differential equation for nonzero spin.
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Thanks for listening!